DEPENDENCE OF NSGA-II HYPERPARAMETERS ON MODEL PARAMETER VARIATIONS IN LQR CONTROLLER TUNING

TOMAS HOLOUBEK[1]*, PAVEL SKRABANEK[1]
[1]Faculty of Mechanical Engineering, Brno University of Technology, Brno, Czech Republic
DOI: 10.17973/MMSJ.2026_06_2025171
***tomas.holoubek@vutbr.cz**

ABSTRACT

This paper investigates the relationship between the physical properties of a controlled dynamic system and the optimal configuration of hyperparameters in the multi-objective evolutionary algorithm NSGA-II (Non-dominated Sorting Genetic Algorithm II). The aim is to determine whether, and to what extent, the optimal hyperparameter settings of NSGA-II change in response to physical modifications of the system. The test model is a rotary inverted pendulum, whose physical characteristics are altered by shifting the position of its center of mass. For each tested center-of-mass position, NSGA-II generates a Pareto front of optimal settings of a linear quadratic regulator with respect to maximum overshoot and settling time of the controlled system. The search for optimal hyperparameters for each configuration is performed using Bayesian optimization. Experimental results show that some hyperparameters exhibit weak dependence on the physical changes in the model, while others remain stable. These findings may serve as a basis for establishing general guidelines for effective tuning of evolutionary algorithms depending on the characteristics of the controlled dynamic system.

KEYWORDS: hyperparameters, evolutionary algorithm, Non-dominated Sorting Genetic Algorithm II, NSGA-II, Bayesian optimization, linear quadratic regulator, LQR, Furuta pendulum

1 INTRODUCTION

Control systems can be implemented using various types of controllers. For *Linear Time-Invariant* (LTI) systems formulated in the state-space representation, the *Linear Quadratic Regulator* (LQR) is a widely adopted and well-established standard. Unlike classical controllers that operate primarily on the control error (such as PID), LQR uses all system states, making it particularly suitable for systems with a higher number of states (higher-dimensional state space).

Tuning the LQR controllers is carried out by selecting the weighting matrices **Q** and **R**, which penalize the system states and the control effort, respectively. This allows the designer to reflect different performance requirements, such as settling time or maximum overshoot. Since no universal tuning procedure exists, these weights are typically chosen empirically [Masti 2021] [Svarc 2011], although modern approaches based on optimization methods have also been proposed.

The optimization methods used for LQR tuning can be classified into single-objective and multi-objective function methods. Particularly useful are evolutionary algorithms. Whereas optimization methods that yield only a single solution are known as *Single-Objective Evolutionary Algorithms* (SOEA), *Multi-Objective Evolutionary Algorithms* (MOEA) search for an entire set of mutually non-dominated solutions. The drawback of SOEA approaches is the need to merge multiple criteria, typically by assigning weights, leading to a single scalar objective. In contrast, MOEAs produce an entire Pareto front of optimal trade-off solutions.

From the perspective of LQR controller tuning, MOEAs make it possible to obtain a set of Pareto-optimal controller settings with respect to the most common regulation criteria: the settling time T_s and the maximum output deviation $y_{\max}$. The visualization of the Pareto front provides a comprehensive overview from which one can easily and repeatedly selected an LQR controller according user preferences (Fig. 1).

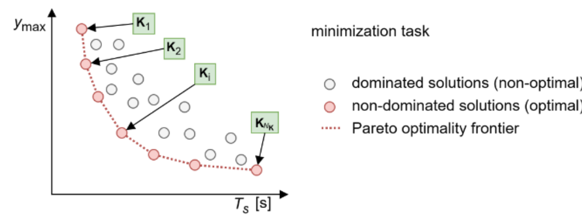

Figure 1. Pareto front of LQR controller settings with respect to the settling time T_s and the maximum output deviation $y_{\max}$. Each point on the front is associated with the corresponding controller gain matrix **K** [Holoubek 2025].

Typical representatives of MOEA are the *Non-Dominated Sorting Genetic Algorithm II* (NSGA-II) [Deb 2002] [Rahimi 2023] [Verma 2021], *Multi-Objective Particle Swarm Optimization* [Alvarez 2005] [Hlal 2019] and t*he third evolution step of generalized differential evolution* [Kukkonen 2005] [Quresh 2019]. A suitable tool for generating the Pareto-optimal controller settings is NSGA-II, the high-performance MOEA designed for problems with conflicting objectives. It evaluates solutions based on dominance and fast non-dominated sorting, employs elitism to preserve the best individuals, and uses crowding distance to maintain population diversity. The resulting Pareto front thus represents a set of high-quality trade-off solutions.

Even when using NSGA-II, generating Pareto-optimal controller settings remains computationally intensive. It is also important to note that the obtained settings are optimal only for the specific system and operating conditions under which the optimization was performed. Consequently, even a slight modification of the system (e.g., changes in model parameters) requires the entire optimization process to be executed again in order to obtain a valid Pareto front.

One way to influence the time required to obtain a solution from NSGA-II is by adjusting its hyperparameters (the parameters of the optimization method). Their choice affects the speed of convergence as well as the quality of the results and the stability of the algorithm. Identifying suitable hyperparameter values is challenging because their influence on convergence is typically nonlinear and may involve strong interactions.

Existing research on hyperparameter optimization appears predominantly in the context of neural networks and machine learning methods (EA) [Soares 2025], [Wang 2025]. In the field of evolutionary algorithms, prior work mainly addresses general hyperparameter tuning and performance comparison [Roman

2016], [Ojha 2022], or focuses on optimizing controller parameters for specific physical models [Solis 2025]. Other studies analyze optimization on fixed control systems [Zhou 2022]. However, no available study investigates how the *optimal* EA hyperparameters depend on changes in the parameters of a controlled system.

To study how changes of the system parameters influence the optimal setting of NSGA-II hyperparameters by LQR controller tuning, the Furuta rotary inverted pendulum was chosen as a representative test system [Cazzolato 2011]. The parameter modification is introduced by shifting the center of mass of the pendulum link. The dependence of optimal setting of the hyperparameters on the pendulum's center-of-mass position is subsequently examined using statistical methods.

To search for optimal hyperparameter setting, various meta-optimization approaches can be employed. Well-known methods such as *Grid search* or *Random search* [Bergstra 2012] are appropriate for small tasks; however, their computational and memory demands grow rapidly with the size of the search space and the ranges of the parameters. As a result, the computational cost can quickly become prohibitive or practically infeasible. An alternative is the use of evolutionary algorithms *such as Genetic Algorithm* [Shanthi 2022], *Particle Swarm Optimization* [Daviran 2025], and *Differential Evolution* [Nikravesh 2024], which have been successfully applied to hyperparameter optimization. However, for complex tasks, *Bayesian Optimization* (BO) is often preferred [Bergstra 2011]. It is well suited for finding optimal hyperparameters of computationally expensive algorithms in situations where the gradient of the objective function is unavailable. For this reason, BO is used in this study for identifying the optimal hyperparameter values of the NSGA-II algorithm.

2 MATERIALS AND METHODS

State space model

The synthesis of an LQR controller is based on a state-space formulation of the system dynamics, given by:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \qquad (1)$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t).$$

where $\mathbf{x}(t)$ is the state vector, $\dot{\mathbf{x}}(t)$ its time derivative, $\mathbf{u}(t)$ is the control input, and $\mathbf{y}(t)$ is the system output. The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ represent the state, input, output, and feedthrough dynamics, respectively.

LQR controller

The LQR is applied using an LTI approximation around the equilibrium point. The control input is defined as:

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t), \qquad (2)$$

where $\mathbf{K} \in \mathbb{R}^{m\times n}$ is the feedback gain matrix minimizing the quadratic cost function:

$$J = \int_0^\infty \left(\mathbf{x}(t)^\top \mathbf{Q}\,\mathbf{x}(t) + \mathbf{u}(t)^\top \mathbf{R}\,\mathbf{u}(t)\right) dt. \qquad (3)$$

The matrix $\mathbf{Q} \in \mathbb{R}^{n\times n}$ penalizes the deviation of individual states and must be symmetric and positive semidefinite. The matrix $\mathbf{R} \in \mathbb{R}^{m\times m}$ penalizes the control input and must be symmetric and positive definite. Here, *m* and *n* denote the number of system inputs and states, respectively. The optimal gain $\mathbf{K}$ is obtained by solving the algebraic *Riccati* equation. In practical implementation, it is necessary to include actuator saturation (e.g. voltage limits) when evaluating the controller performance.

Non-Dominated Sorting Genetic Algorithm II

NSGA-II is an evolutionary multi-objective optimization algorithm. Each objective is expressed as a single fitness function, and minimization of fitness is assumed. The algorithm operates on a population of individuals, where each individual encodes one candidate solution. The performance of NSGA-II is influenced by a number of parameters, most notably the population size, number of generations, crossover probability, and mutation probability. Also, other parameters such as crossover distribution index η_c or mutation distribution index η_m may influence the performance [Deb 2002].

LQR Parameterization

For the controller tuning, only the diagonal elements of the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ are considered. This choice significantly reduces the dimensionality of the optimization problem and reflects a common assumption that the states and control inputs are penalized independently. Off-diagonal terms, which represent cross-couplings between states or inputs, are therefore omitted.

Encoding of individuals for LQR tuning

Each individual is formed by the decision variables corresponding to the diagonal entries of the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$:

$$\mathbf{z} = [q_{x_1} \ \cdots \ q_{x_n} \ \ r_{u_1} \ \cdots \ r_{u_m}], \qquad (4)$$

where q_{x_i} denotes the i-th diagonal element of the matrix $\mathbf{Q}$, penalizing the state x_i, and r_{u_i} denotes the i-th diagonal element of the matrix $\mathbf{R}$, penalizing the control input u_i.

Fitness functions for LQR tuning

The most important criteria in controller design are the settling time T_s and the maximum output deviation $y_{\max}$. Thus, the fitness functions to be minimized by NSGA-II are:

$$y_{\max} = \max_{y\in \boldsymbol{y}} \max_{t\in[0,t_s]} |y(t)|, \qquad (5)$$

$$T_s = \max_{y\in \boldsymbol{y}} \min_{t\in\{0,t_s\}} \{t \mid \ |y(\tau) - y(t_s)| \le \varepsilon |y(t_s) - y(0)|, \forall\tau \in [t, t_s]\}, \qquad (6)$$

where ε denotes the tolerance band of the steady-state value.

Bayesian Optimization

BO is a global optimization method designed for expensive black-box functions. Its key parameters include the choice of surrogate model and the acquisition function, which controls the balance between exploration and exploitation. In this study, a Gaussian process is used as the surrogate model and Expected Improvement Plus as the acquisition function.

Objective function for BO

BO is used for the tuning of the NSGA-II hyperparameters. The standard measure for NSGA-II evaluation is the hypervolume indicator HV [Bradstreet 2011] [Zitzler 2007]. To evaluate the quality of a set of nondominated solutions on the Pareto front produced by NSGA-II for LQR tuning, the indicator is computed for the Pareto-front points sorted according to T_s as:

$$HV = \sum_{i=1}^{N} \left(x_{\text{ref}} - T_{\text{s}}^{(i)} \right) x_i, \qquad (7)$$

$$x_{\text{i}} = \begin{cases} y_{\text{ref}} - y_{\max}^{(1)}, & i = 1 \\ y_{\max}^{(i-1)} - y_{\max}^{(i)}, & i > 1 \end{cases} \qquad (8)$$

where the reference point is defined as:

$$x_{\text{ref}} = \max_i T_s^{(i)} + x_{\text{shift}}, \quad x_{\text{shift}} > 0$$
$$y_{\text{ref}} = \max_i y_{\max}^{(i)} + y_{\text{shift}}, \quad y_{\text{shift}} > 0 \qquad (9)$$

The values $x_{\text{shift}}, y_{\text{shift}}$ represent a non-zero offset (“margin”) that moves the reference point slightly beyond the worst obtained solution (Fig. 2). This offset ensures a correct and numerically stable computation of HV.

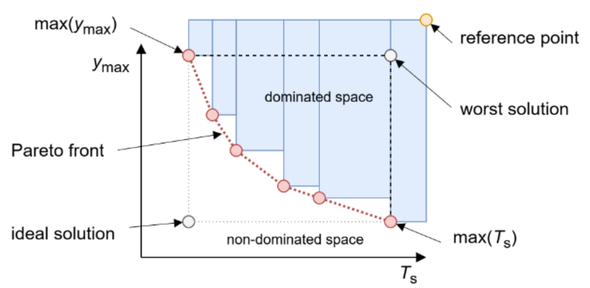

Figure 2. Hypervolume indicator for a minimization problem.

Since a larger hypervolume corresponds to a better Pareto-front approximation, the Bayesian optimization objective needs to be minimized by using the negative hypervolume, i.e.:

$$J = -HV. \qquad (10)$$

Controlled system

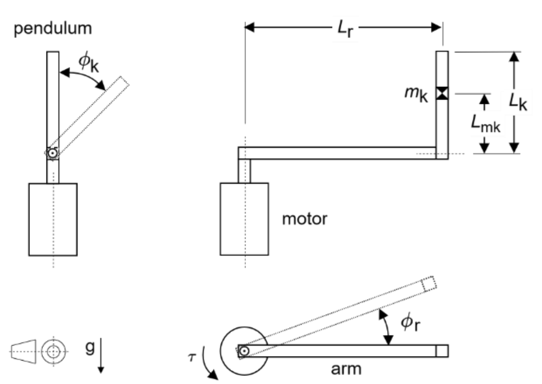

Figure 3. Technical drawing of a Furuta pendulum with used symbols.

The system to be controlled by the LQR controller is the Furuta pendulum. It consists of a horizontal arm driven by an electric motor and a pendulum attached perpendicularly at its end, rotating freely in the vertical plane (Fig. 3). The control task is stabilization of the pendulum in the upright equilibrium position. Control actions can be performed by adjusting the motor voltage $U(t)$. The controlled variables are the angle of the arm and the pendulum, $\phi_r(t)$ and $\phi_k(t)$, respectively. The other dynamic variables are the motor current $i(t)$, and the angular velocity of the arm and the pendulum, $\dot{\phi}_r(t)$ and $\dot{\phi}_k(t)$, respectively. The system parameters and initial conditions are summarized in Tab. 1.

Table 1. Model parameters and initial conditions.

Symbol	Value	Description
m_{k}	0.16 kg	Pendulum mass
L_{r}	0.15 m	Arm length
L_{k}	0.20 m	Pendulum length
L_{mk}	0.10 m	Distance of pendulum’s center of mass from the joint
I_{r}	$9 \cdot 10^{-4}$ kg·m^2	Arm moment of inertia
I_{k}	$2.1 \cdot 10^{-4}$ kg·m^2	Pendulum moment of inertia
g	9.81 m/s^2	Gravitational acceleration
K_{t}	0.044 N·m/A	Motor torque constant
K_{e}	0.079 V·s/rad	Back-EMF constant
L_{mot}	0.01 H	Motor inductance
R_{mot}	$8\ \Omega$	Motor resistance
$\phi_{\text{r}}(0)$	0 rad	Initial arm angle
$\phi_{\text{k}}(0)$	0.524 rad	Initial pendulum angle
$\dot{\phi}_{\text{r}}(0)$	0 rad/s	Initial arm angular velocity
$\dot{\phi}_{\text{k}}(0)$	0 rad/s	Initial pendulum angular velocity
$i(0)$	0 A	Initial current
$U(0)$	0 V	Input voltage

Considering that the LQR controller stabilizes the pendulum in the upright position and that the initial pendulum position is close to this equilibrium, the system dynamics in this regime can be approximated by a model linearized around the upright equilibrium point $\phi_r = \phi_k = \dot{\phi}_r = \dot{\phi}_{\text{k}} = i = 0$ [Holoubek 2025]. In the state-space form (1), the state, input, and output vectors are given by:

$$\mathbf{x}(t) = [\,\phi_{\text{r}}(t),\ \phi_{\text{k}}(t),\ \dot{\phi}_{\text{r}}(t),\ \dot{\phi}_{\text{k}}(t),\ i\,(t)\,]^{\top},$$
$$\mathbf{u}(t) = [\,U(t)\,]^{\top}, \qquad (11)$$
$$\mathbf{y}(t) = [\,\phi_{\text{r}}(t),\ \phi_{\text{k}}(t)\,]^{\top}$$

The state matrix is given by:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & A_{32} & 0 & 0 & A_{35} \\ 0 & A_{42} & 0 & 0 & A_{45} \\ 0 & 0 & A_{53} & 0 & A_{55} \end{bmatrix}, \qquad (12)$$

where

$$\text{den} = (I_r + m_{\text{k}} L_{\text{r}}^2)(I_{\text{k}} + m_{\text{k}} L_{\text{mk}}^2) - (m_{\text{k}} L_{\text{r}} L_{\text{mk}})^2,$$

$$A_{32} = -\frac{m_k^2 L_r L_{mk}^2 g}{\text{den}},$$
$$A_{35} = \frac{(I_k + m_k L_{mk}^2)}{\text{den}} K_t,$$
$$A_{42} = \frac{(I_r + m_k L_r^2) m_k g L_{mk}}{\text{den}},$$
$$A_{45} = -\frac{m_k L_r L_{mk}}{\text{den}} K_t,$$
$$A_{53} = -\frac{K_e}{L_{mot}}, \quad A_{55} = -\frac{R_{mot}}{L_{mot}}.$$

The input matrix is:

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{L_{mot}} \end{bmatrix}^\top. \tag{13}$$

The output and feedthrough matrices are:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{14}$$

Design of the experiment

The goal of the experiment is to obtain data for studying how changes in the system parameters influence the optimal settings of the NSGA-II hyperparameters in the context of LQR controller tuning. The parameter variation is introduced by shifting the pendulum's center of mass L_{mk}. Its nominal position is set at one quarter of the pendulum length measured from the joint, and a total of ten uniformly distributed positions within the interval $[0.05,\ 0.20]$ m are tested. For each position, the initial conditions of the system are $\mathbf{x_0} = (0,\ 0.524,\ 0,\ 0,\ 0)^\top$.

For each defined value of L_{mk}, the NSGA-II is used to generate the corresponding Pareto front of optimal LQR settings. The algorithm minimizes the criteria (5) and (6), taking into account both process outputs (11). The tolerance band of the steady-state value ε is set to 0.05. As follows from (4), the individuals are encoded as

$$\mathbf{z} = [q_{\phi_r} \quad q_{\phi_k} \quad q_{\dot\phi_r} \quad q_{\dot\phi_k} \quad q_i \quad r_U]. \tag{15}$$

The admissible range of the optimized LQR parameters is limited by the physical properties of the model and the capabilities of the actuator. Thus, the upper and lower search limit is set as follows:

$$\mathbf{lb} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.1], \quad \mathbf{ub} = [10 \quad 10 \quad 10 \quad 10 \quad 24 \quad 10]. \tag{16}$$

Due to the stochastic nature of NSGA-II, a single run is insufficient for reliable evaluation. Therefore, 10 independent optimization runs are performed for each center-of-mass position. The number of runs has been chosen with respect to the high computational time required for a single execution of NSGA-II.

For each run, BO is used to search for optimal settings of the NSGA-II hyperparameters. The hyperparameters taken into account and their ranges are as follows:

- population size: [20, 200],
- number of generations: [20, 500],
- mutation probability: [0.01, 0.4],
- crossover probability: [0.5, 1.0],
- crossover distribution index η_c: [5, 50],
- mutation distribution index η_m: [5, 80].

The maximum number of iterations per BO run is set to 50. To reduce the overall duration of the experiment, the hyperparameter optimization is terminated early when the relative improvement of HV falls below 0.01 for 20 consecutive generations. Early stopping in BO is enabled only after at least 6 NSGA-II evaluations have been completed. Additionally, the relative improvement threshold of NSGA-II between successive generations is set to 0.1.

Analysis of hyperparameter dependences

The aim of the analysis is to identify potential dependencies of the NSGA-II hyperparameters on the pendulum center-of-mass position L_{mk}. Since the influence of one variable on another is investigated, tools of univariate analysis are sufficient. To reveal linear dependencies, monotonic trends, and the degree of concordance or discordance between pairs of observations, the Pearson, Spearman, and Kendall correlation coefficients are calculated for each hyperparameter. Considering the relatively small number of samples per L_{mk}, the Kruskal–Wallis test is used to verify whether the observed dependencies are statistically significant. Under the assumption of similarly shaped distributions, the null hypothesis corresponds to the medians of all groups being equal, and it is rejected when the p-value falls below the significance level of 0.05.

3 RESULTS

Correlation Coefficients

The values of the *Pearson*, *Spearman*, and *Kendall* correlation coefficients for the individual hyperparameters are shown in Figures 4 to 6. Numerical values are provided for each hyperparameter. To improve readability, the individual values are color-shaded using a heat map.

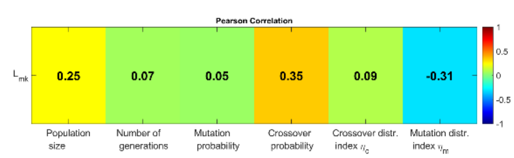

Figure 4. Correlation plot of hyperparameters and center-of-mass shift, Pearson method.

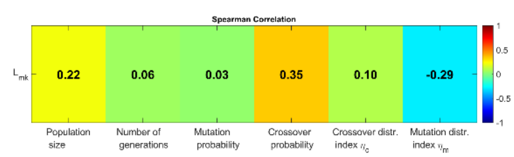

Figure 5. Correlation plot of hyperparameters and center-of-mass shift, Spearman method.

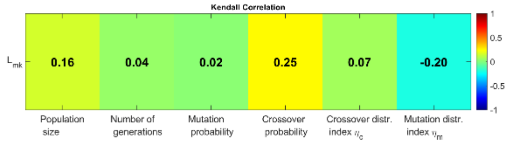

Figure 6. Correlation plot of hyperparameters and center-of-mass shift, Kendall method.

Kruskal–Wallis test

The test results for the individual hyperparameters are presented in Table 2. For each hyperparameter (first column), the corresponding $p-$ value is given (second column), the test outcome for the significance level 0.05 (third column), and whether there is a statistically sufficiently large difference between the medians of the data examining the dependence of NSGA-II hyperparameter values on the change in the pendulum's center-of-mass position (the last column).

Table 2. Results of the Kruskal–Wallis test.

Hyperparameter	p-value	H_0	Statistically significant difference
Population size	0.0180	**Rejected**	**Yes**
Number of generations	0.3103	Not reject.	No
Crossover probability	0.0983	Not reject.	No
Mutation probability	0.7628	Not reject.	No
Crossover distrib. Index η_c	0.6668	Not reject.	No
Mutation distrib. index η_m	0.1092	Not reject.	No

4 DISCUSSION

To determine the dependence, correlations between the hyperparameters and the center-of-mass position were examined using three correlation methods: Pearson (Fig. 4), Spearman (Fig. 5), and Kendall (Fig. 6). Strongest correlations have been observed for the hyperparameter *population size, crossover probability* and *mutation distribution index* η_m. While *population size* and *crossover probability* show weak to moderate positive correlation, η_m shows an approximately equally strong negative correlation. A certain degree of dependence is therefore observable, but it is not convincing, because the test results are limited by the small dataset, and the values do not exceed the threshold of 0.5.

The Kruskal–Wallis test was carried out to identify statistically significant differences between the medians of one group of data (the center-of-mass positions). Focusing on the three hyperparameters for which a certain degree of dependence was presumed based on the correlation analysis, the result shows a statistically sufficiently large difference for the hyperparameter population (Table 2, first row, last column), where the null hypothesis H_0 was rejected. The crossover probability hyperparameter also exhibits a notably low p-value and behaves very similarly to what was observed in the previous correlation test. The mutation distribution index η_m also approached the decision boundary of $p = 0.05$. These hyperparameters should certainly be included in future, more detailed investigations, as the test confirmed that the observed correlation is not random but appears to be systematic. This means that the differences observed in these hyperparameters show sufficiently large variations across the different center-of-mass positions of the pendulum, thereby confirming the examined dependence.

The remaining hyperparameters (number of generations, mutation probability, and crossover distribution index η_c) remained relatively constant throughout the changes in the center-of-mass position and did not exhibit any dependence. This means that for the same system, they could be set to a constant value regardless of what change occurs in the center-of-mass position.

From a practical standpoint, the insensitivity of some hyperparameters to changes in the parameters of the controlled system offers an opportunity to reduce computational demands and simplify the tuning process. Hyperparameters that exhibit no dependence can be fixed at predefined values and omitted from tuning. For the remaining hyperparameters, recommended ranges can be specified without the risk of under-sizing or over-sizing. The restriction of their ranges would further speed up the hyperparameter optimization, and thus the tuning of LQR controllers using NSGA-II for various system parameters.

The experiment was conducted on a single system with a specific change in the center-of-mass position. To generalize the results and achieve a better understanding of hyperparameter behavior, broader studies on various other types of models are required, while also reinforcing statistical robustness through repeated trials and appropriate evaluation metrics. The investigation should continue with simple systems in which, for example, the influence of other mechanical components is minimized, and then gradually transition to more complex ones, accompanied by systematic analyses aimed at identifying stable hyperparameter settings. This combined approach would support the broader applicability and transferability of the proposed methodology in real-world settings.

5 CONCLUSION

The research presented in this article has explored an understudied aspect of hyperparameter selection for NSGA-II in the context of LQR controller tuning. The proposed experiment did not rule out the possibility of identifying fixed settings for certain hyperparameters that could remain applicable across models with modified parameters, or even across whole classes of models. These findings suggest that further investigation into hyperparameter robustness and transferability is warranted. Identifying hyperparameters that admit fixed settings across families of controlled systems would significantly accelerate the design of LQR controllers using NSGA-II. Moreover, Pareto fronts of optimal LQR controller settings with respect to settling time and maximum overshoot would greatly facilitate the deployment of LQR controllers in real-world applications. Altogether, the results of the present study, along with future research, may help close an existing knowledge gap and contribute to more efficient and reliable controller tuning methodologies.

REFERENCES

[Alvarez 2005] Alvarez-Benitez, J. E., Everson, R. M. and Fieldsend, J. E. A MOPSO Algorithm Based Exclusively on Pareto Dominance Concepts. In: *Evolutionary Multi-Criterion Optimization: Third International Conference, EMO 2005*, Springer, pp. 459–473, 2005.

[Bergstra 2011] Bergstra, J., Bardenet, R., Bengio, Y. and Kegl, B. Algorithms for Hyper-Parameter Optimization.

Advances in Neural Information Processing Systems, vol. 24, 2011.

[Bergstra 2012] Bergstra, J. and Bengio, Y. Random search for hyper-parameter optimization. *The Journal of Machine Learning Research*, 2012, vol. 13, no. 1, pp. 281–305.

[Bradstreet 2011] Bradstreet, L. *The Hypervolume Indicator for Multi-Objective Optimisation: Calculation and Use.* University of Western Australia Perth, 2011.

[Cazzolato 2011] Cazzolato, B. S. and Prime, Z. On the Dynamics of the Furuta Pendulum. Online, 2011. Available at: https://digital.library.adelaide.edu.au/dspace/bitstream/2440/66777/1/hdl_66777.pdf.

[Daviran 2025] Daviran, M., Maghsoudi, A. and Ghezelbash, R. Optimized AI-MPM: Application of PSO for tuning the hyperparameters of SVM and RF algorithms. *Computers & Geosciences*, 2025, vol. 195, p. 105785.

[Deb 2002] Deb, K., Pratap, A., Agarwal, S. and Meyarivan, T. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002. doi: 10.1109/4235.996017.

[Fisher 1925] Fisher, R. A. *Statistical Methods for Research Workers*. 1st ed. Edinburgh: Oliver & Boyd, 1925. Classic work in statistics.

[Hlal 2019] Hlal, M. I., Ramachandaramurthy, V. K., Padmanaban, S., Kaboli, H. R., Pouryekta, A., Bin, T. A. R. and Abdullah, T. NSGA-II and MOPSO Based Optimization for Sizing of Hybrid PV/Wind/Battery Energy Storage System. *International Journal of Power Electronics and Drive Systems*, vol. 10, no. 1, pp. 463–470, 2019.

[Holoubek 2025] Holoubek, T. and Skrabanek, P. *Genetic Optimization Meets Visualization: Multi-objective LQR Tuning for the Furuta Pendulum.* In: Proceedings of the International Conference on Electric Drives Production and Energy (EDPE), Dubrovnik, Croatia, September 2025. IEEE, pp. 1–5. doi: 10.1109/EDPE66853.2025.11224248.

[Kukkonen 2005] Kukkonen, S. and Lampinen, J. GDE3: The Third Evolution Step of Generalized Differential Evolution. In: *2005 IEEE Congress on Evolutionary Computation (CEC)*, vol. 1, pp. 443–450, 2005. IEEE. doi: 10.1109/CEC.2005.1554717.

[Masti 2021] Masti, D., Zanon, M. and Bemporad, A. *Tuning LQR controllers: A sensitivity-based approach*. IEEE Control Systems Letters, Vol. 6, pp. 932–937, 2021.

[Nikravesh 2024] Nikravesh, N. and Keyvanpour, M. R. Parameter tuning for software fault prediction with different variants of differential evolution. *Expert Systems with Applications*, 2024, vol. 237, p. 121251.

[Ojha 2022] Ojha, V., Timmis, J. and Nicosia, G. Assessing Ranking and Effectiveness of Evolutionary Algorithm Hyperparameters Using Global Sensitivity Analysis Methodologies. Swarm and Evolutionary Computation, vol. 74, p. 101130, 2022. doi: 10.1016/j.swevo.2022.101130.

[Quresh 2019] Quresh, K., Rahnamayan, S., He, Y. and Liscano, R. Enhancing LQR Controller Using Optimized Real-Time System by GDE3 and NSGA-II Algorithms and Comparing with Conventional Method. In: *2019 IEEE Congress on Evolutionary Computation (CEC)*, pp. 2074–2081, 2019. IEEE. doi: 10.1109/CEC.2019.8790071.

[Rahimi 2023] Rahimi, I., Gandomi, A. H., Nikoo, M. R. and Chen, F. A Comparative Study on Evolutionary Multi-Objective Algorithms for Next Release Problem. *Applied Soft Computing*, vol. 144, p. 110472, 2023.

[Roman 2016] Roman, I., Ceberio, J., Mendiburu, A. and Lozano, J. A. Bayesian Optimization for Parameter Tuning in Evolutionary Algorithms. In: *2016 IEEE Congress on Evolutionary Computation (CEC)*, pp. 4839–4845, 2016. doi: 10.1109/CEC.2016.7744410.

[Shanthi 2022] Shanthi, D. L. and Chethan, N. Genetic algorithm based hyper-parameter tuning to improve the performance of machine learning models. *SN Computer Science*, 2022, vol. 4, no. 2, p. 119.

[Soares 2025] Soares, R. C., de Oliveira, A. L. and Baccarini, L. M. Integration of Bayesian optimization into hyperparameter tuning of the particle swarm optimization algorithm to enhance neural network-based bearing failure detection. *Measurement*, 2025, art. 115829. doi: 10.1016/j.measurement.2024.115829.

[Solis 2025] Solis, M. A., Thomas, S. S. and Choque-Surco, C. A. *Bayesian Optimization for the Synthesis of Generalized State-Feedback Controllers in Underactuated Systems.* Mathematics, 13(19), 3139, 2025. DOI: 10.3390/math13193139.

[Svarc 2011] Svarc, I., Matousek, R., Seda, M. and Viteckova, M. *Automaticke rizeni.* Brno: CERM – Akademicke nakladatelstvi, 2011. (in Czech).

[Verma 2021] Verma, S., Pant, M. and Snasel, V. A Comprehensive Review on NSGA-II for Multi-Objective Combinatorial Optimization Problems. *IEEE Access*, vol. 9, pp. 57757–57791, 2021.

[Wang 2025] Wang, R., Cheung, C. F., Zang, Y., Wang, C. and Liu, C. Material Removal Rate Optimization with Bayesian-Optimized Differential Evolution Based on Deep Learning in Robotic Polishing. *Journal of Manufacturing Systems*, vol. 78, pp. 178–186, 2025. doi: 10.1016/j.jmsy.2024.11.014.

[Zhou 2022] Zhou, Y., Sun, Y., Wang, S. and Huang, X. Hyperparameter Optimization of Evolutionary Control Algorithms via Surrogate-Assisted Bayesian Learning. Engineering Applications of Artificial Intelligence, vol.

109, p. 104653, 2022. doi: 10.1016/j.engappai.2021.104653.

[Zitzler 2007] Zitzler, E., Brockhoff, D. and Thiele, L. The Hypervolume Indicator Revisited: On the Design of Pareto-Compliant Indicators via Weighted Integration. In: *Evolutionary Multi-Criterion Optimization: 4th International Conference, EMO 2007, Matsushima, Japan, March 5–8, 2007. Proceedings 4*, Springer, 2007, pp. 862–876.

CONTACTS:

Ing. Tomas Holoubek
Brno University of Technology,
Faculty of Mechanical Engineering,
Technicka 2, 616 69 Brno,
Czech Republic
E-mail: tomas.holoubek@vutbr.cz
Website: https://www.vutbr.cz

doc. Ing. Pavel Skrabanek, Ph.D.
Brno University of Technology,
Faculty of Mechanical Engineering,
Technicka 2, 616 69 Brno,
Czech Republic
E-mail: skrabanek@fme.vutbr.cz
Website: https://www.vutbr.cz

LICENSE CREATIVE COMMONS: