A MODAL EXCITATION SPECTRUM-BASED FRAMEWORK FOR VIBRATION-ORIENTED OPTIMIZATION OF CYCLICALLY SYMMETRIC BLADED STRUCTURES

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The vibration-oriented optimization of cyclically symmetric bladed structures subjected to aerodynamic excitation remains computationally demanding because conventional strategies typically require repeated harmonic response analyses throughout the optimization process. Controlled modifications of blade geometry, mass or stiffness can reduce critical resonant responses; however, the most favourable modification depends on the operating excitation, the structural dynamic characteristics and practical design constraints. This paper presents a computationally efficient optimization framework based on the proposed Modal Excitation Spectrum (MES). Unsteady aerodynamic pressure fields obtained from computational fluid dynamics (CFD) are transformed into equivalent harmonic nodal forces acting on the structural finite element model. These forces are projected onto mass-normalized structural mode shapes to obtain generalized modal forces representing the aerodynamic excitation in the modal domain. The MES describes the distribution of modal excitation over the selected structural modal branches, whose natural frequencies are tracked relative to the prescribed operating-frequency range. The maximum mass-normalized generalized modal force among the selected structural modal branches is employed as the objective function. Consequently, repeated harmonic response analyses are excluded from the optimization loop, thereby reducing the forced-response computation required during optimization. A conventional harmonic response analysis is performed only after convergence to independently verify the vibration response and the maximum equivalent stress over the harmonic phase cycle of the optimized design. The framework permits independent blade design variables and can accommodate geometric, material, mass and stiffness modifications together with engineering constraints such as rotor balance. It therefore admits both uniform tuning changes and blade-specific intentional mistuning without prescribing either type of final configuration. The methodology is demonstrated on a thirteen-bladed turbocharger turbine wheel for a large marine engine. Although the blade-tip variables were independent, the balance-constrained optimization converged to an identical modification of all blade tips. The final harmonic response analysis confirmed a substantial reduction in the maximum equivalent stress over the harmonic phase cycle within the investigated operating-speed interval.

KEYWORDS

modal excitation spectrum; bladed structures; structural optimization; intentional mistuning; modal tuning; rotor balance; turbomachinery

1 INTRODUCTION

Cyclically symmetric bladed structures are widely used in turbomachinery because they provide high structural efficiency, low weight and favourable aerodynamic performance [Dixon 2014, Cumpsty 2006]. Typical applications include turbocharger turbine wheels, compressor impellers, axial compressors, axial turbines and integrally bladed disks [Dixon and Hall 2014]. Owing to their periodic geometry, these structures exhibit well-defined dynamic characteristics that can be efficiently described using cyclic symmetry theory [Thomas 1979, Meirovitch 1986]. However, periodic aerodynamic loading may strongly excite the corresponding structural modes when an excitation frequency approaches a natural frequency [Whitehead 1966, Griffin 1984]. In automotive applications, the development of efficient and reliable turbocharging systems forms part of broader efforts to improve engine performance and reduce exhaust emissions. Complementary fuel- and combustion-oriented approaches include the use of oxygenated gasoline additives [Cakmak 2018] and hydroxy-gas enrichment of lean-operated LPG-fuelled spark-ignition engines [Cakmak 2021].

Unsteady aerodynamic forces generated by rotor-stator interaction, wake impingement and pressure fluctuations can produce high vibration amplitudes and dynamic stresses [Denton 1993, Pan 2021, Hassan 2019]. In high-speed turbomachinery, repeated near-resonant loading may reduce fatigue life and ultimately lead to structural failure [Whitehead 1966, Denton 1993]. Accurate prediction and effective mitigation of forced vibration therefore remain fundamental objectives in turbomachinery design [Dixon 2014, Rao 2017].

One possible mitigation strategy is to modify the dynamic properties of the blades. Uniform modifications preserve cyclic symmetry and change the overall tuning of the wheel, whereas controlled blade-to-blade variations introduce intentional mistuning [Castanier 2006, Petrov 2010]. Intentional mistuning can split repeated natural frequencies, alter mode localization and redistribute the forced response [Feiner and Griffin 2002, Han 2014, Pistek 2020]. Its effect is not intrinsically beneficial, however, because the resulting response depends on the mistuning pattern, the aerodynamic excitation and the operating range. The most favourable structural modification must therefore be identified through optimization.

Despite substantial progress in the analysis of tuned and mistuned bladed structures, the practical optimization of blade modifications remains computationally demanding [Petrov 2010, Han 2014]. Existing procedures typically require a forced response or harmonic response analysis for every candidate design. Because each modification changes the natural frequencies and mode shapes, the complete dynamic response must be recomputed repeatedly. For realistic finite element models and large design spaces, these repeated analyses usually dominate the computational cost.

Most existing methods formulate the optimization problem directly in the structural response domain. Repeated forced-response analyses therefore remain an inherent part of the optimization loop, even though the final response is strongly influenced by the efficiency with which the aerodynamic loading excites the structural modes. This observation motivates a reformulation of the problem in the modal excitation domain.

The present work introduces a computationally efficient optimization framework based on a modal representation of

aerodynamic excitation. The aerodynamic loading is transformed into mass-normalized generalized modal forces, and the Modal Excitation Spectrum (MES) is introduced as a compact description of the excitation of the selected structural modal branches. Formulating the objective function directly in terms of the MES eliminates repeated harmonic response analyses from the optimization loop while retaining the physical relationship between the aerodynamic loading and the structural dynamic properties of each candidate design.

The principal contributions of this work can be summarized as follows:

- a general methodology for transforming CFD-derived aerodynamic excitation into mass-normalized generalized modal forces,
- introduction of the Modal Excitation Spectrum as a unified description of the excitation of selected structural modal branches,
- formulation of a constrained optimization problem based on the maximum modal excitation and independent structural design variables,
- elimination of repeated harmonic response analyses from the optimization loop,
- independent verification of the optimized design using a single conventional harmonic response analysis.

The proposed methodology is generally applicable to vibration-oriented optimization of cyclically symmetric bladed structures. Its capabilities are demonstrated using a turbocharger turbine wheel as a representative engineering application. The framework is independent of the selected optimization algorithm and of the physical realization of the design variables, which may represent uniform modifications, blade-specific variations or a combination of both.

2 THEORETICAL FOUNDATIONS

2.1 Cyclic Symmetry of Bladed Structures

Cyclically symmetric bladed structures constitute one of the most common structural configurations in turbomachinery [Thomas 1979, Meirovitch 1986]. Typical examples include turbocharger turbine wheels, compressor impellers, axial compressor stages, axial turbines and integrally bladed disks (blisks). Their periodic geometry provides favourable structural and aerodynamic characteristics while simultaneously enabling an efficient mathematical description of their dynamic behaviour. Owing to cyclic symmetry, the vibration characteristics of these structures can be represented by a single-sector model together with appropriate cyclic boundary conditions, significantly reducing the computational effort required for modal analysis [Thomas 1979, Bathe 1996].

The free vibration of a cyclically symmetric structure is governed by the generalized eigenvalue problem

$$\boldsymbol{K\phi} = \lambda \boldsymbol{M\phi} \tag{1}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$ denote the global stiffness and mass matrices, respectively, $\boldsymbol{\phi}$ represents the eigenvector associated with the vibration mode and $\lambda = \Omega^2$ is the corresponding eigenvalue. The resulting vibration modes are commonly described using the nodal diameter concept, in which each mode is characterized by a specific circumferential wave number [Thomas 1979, Meirovitch 1986]. This representation provides a natural physical interpretation of the deformation pattern and forms the basis for analysing the dynamic behaviour of cyclically symmetric structures subjected to harmonic excitation.

2.2 Aerodynamic Excitation

During operation, these structural modes are continuously excited by unsteady aerodynamic forces generated by complex flow phenomena within the turbomachine [Denton 1993, Dixon 2014]. Rotor–stator interaction, wake impingement, pressure fluctuations and other periodic flow mechanisms produce harmonic pressure fields acting on the blade surfaces [Denton 1993, Pan 2021]. Such excitation is commonly described in the frequency domain by means of complex pressure amplitudes corresponding to individual excitation orders. Modern computational fluid dynamics (CFD) techniques enable these unsteady pressure distributions to be predicted with high spatial resolution, providing a realistic description of the aerodynamic loading acting on the structure [Pan 2021, Hassan 2019].

If the excitation frequency approaches one of the natural frequencies of the bladed structure, resonance may occur, resulting in a considerable increase of vibration amplitudes and dynamic stresses [Whitehead 1966, Denton 1993]. In practical applications, repeated resonant loading may significantly reduce fatigue life and eventually lead to structural failure. Consequently, the prediction and mitigation of forced vibration remain among the principal objectives in the structural design of turbomachinery components.

2.3 Structural Tuning, Mistuning and Mode Localization

A nominally cyclic structure can be modified either uniformly or non-uniformly. When the same change is applied to every sector, cyclic symmetry is preserved and the modification changes the overall tuning of the structure. When the blade properties differ around the circumference, cyclic symmetry is broken and the structure becomes mistuned [Ewins 1969, Griffin 1984, Petrov 2002, Castanier 2006]. Such differences may arise randomly from manufacturing variability or may be introduced deliberately as intentional mistuning [Petrov 2003].

Mistuning removes the modal degeneracy of the tuned system, splits repeated natural frequencies and may produce mode localization [Hodges 1982, Feiner 2002, Mignolet 1998, Yang 2001]. Random mistuning can amplify the maximum blade response [Mignolet 2001], whereas a carefully optimized intentional pattern can redistribute the response and reduce selected resonance peaks [Crawley 1985, Han 2014]. The resulting effect depends on the spatial pattern of the modifications, the excitation order and the operating frequency range. Figure 1 illustrates a possible beneficial case of intentional mistuning. Within the present framework, intentional mistuning is treated as one admissible subset of a broader structural optimization problem rather than as a prescribed final configuration.

2.4 Modal Representation of Structural Response

The dynamic response of bladed structures is conveniently analysed in modal coordinates using the modal transformation

$$\boldsymbol{u} = \boldsymbol{\Phi q} \tag{2}$$

where $\boldsymbol{u}$ denotes the physical displacement vector, $\boldsymbol{\Phi}$ is the modal matrix and $\boldsymbol{q}$ represents the vector of generalized modal coordinates. This transformation maps the structural response from the physical domain into the modal domain [Meirovitch 1986, Rao 2017]. In this representation, the influence of the external excitation on individual vibration modes can be analysed independently, providing valuable physical insight into the excitation mechanisms governing the structural response.

The contribution of the external loading to each vibration mode is described by the generalized modal force

$$\boldsymbol{Q} = \boldsymbol{\Phi}^T \boldsymbol{F} \tag{3}$$

where $\boldsymbol{F}$ is the external force vector and $\boldsymbol{\Phi}^T$ denotes the transpose of the real modal matrix [He and Fu 2001].

The generalized modal forces quantify the effectiveness with which the external loading excites the individual structural modes and constitute a fundamental quantity in modal analysis.

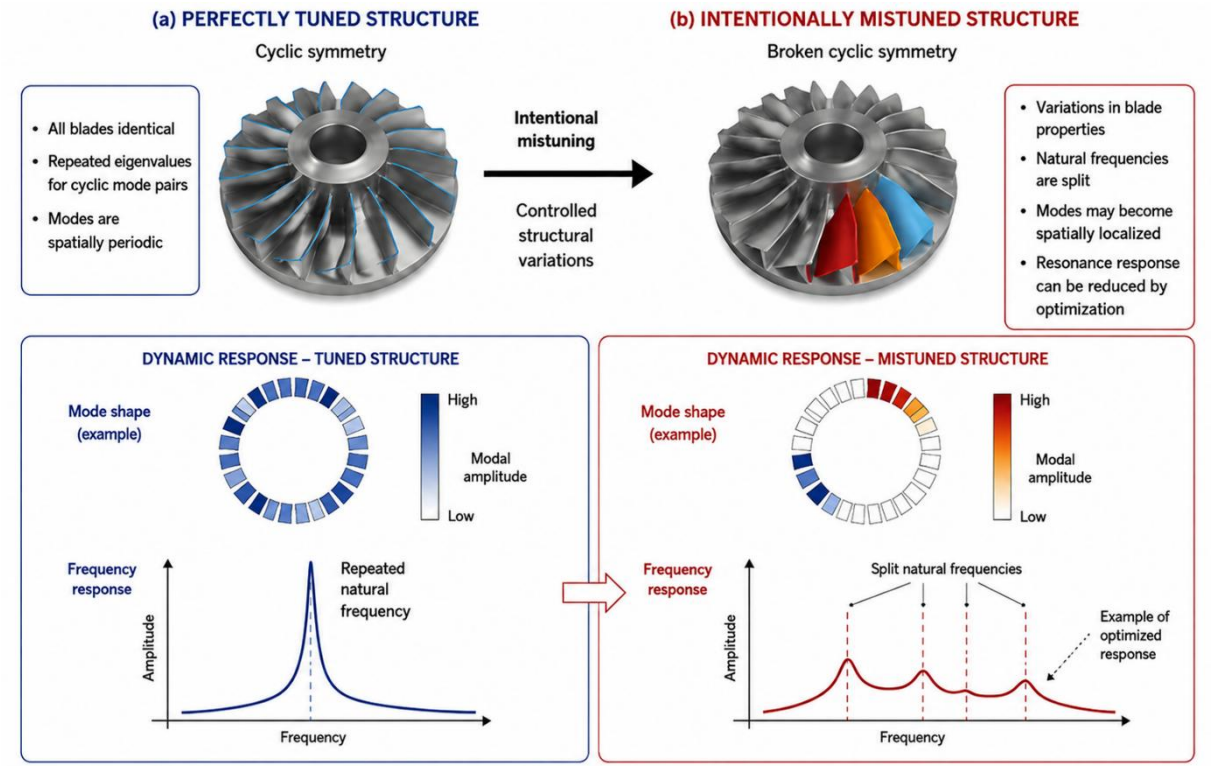


Figure 1. Schematic illustration of a possible beneficial effect of intentional mistuning on a cyclically symmetric bladed structure. Controlled blade-to-blade variations split repeated natural frequencies, alter mode localization and redistribute the forced-response peaks

These theoretical concepts provide the basis for reformulating the optimization problem in the modal excitation domain. The following section introduces the Modal Excitation Spectrum and employs it to evaluate candidate structural modifications without repeated forced-response analyses.

3 PROPOSED MES-BASED OPTIMIZATION FRAMEWORK

The proposed framework transfers vibration-oriented optimization from the structural response domain to the modal excitation domain. Instead of repeatedly evaluating the forced response for every candidate design, the methodology operates directly on the modal representation of the aerodynamic excitation. A harmonic response analysis is performed only after convergence and serves as an independent verification of the optimized design.

The framework integrates computational fluid dynamics, finite element structural analysis and modal analysis into a unified optimization procedure. It is independent of the selected optimization algorithm and of the physical form of the structural modification. Candidate designs may preserve cyclic symmetry or may introduce blade-specific intentional mistuning, depending on the design variables and constraints.

3.1 Computational Workflow

The proposed framework comprises three computational domains followed by a final verification stage. The physical domain provides the aerodynamic excitation, whereas the structural model, modal basis and MES are updated iteratively as the design vector changes. This organization retains the relationship between the aerodynamic loading and the structural dynamic behaviour of each candidate design.

The first computational domain is the physical domain, in which the aerodynamic excitation is obtained from unsteady CFD simulations [Denton 1993, Pan 2021]. The resulting complex pressure distributions on the blade surfaces are transformed into equivalent harmonic nodal forces compatible with the structural finite element model [Hassan 2019, Sato 2014]. The excitation therefore retains its spatial distribution and frequency content. The complete workflow is illustrated in Figure 2.

The second computational domain is the modal domain. For each candidate design, the structural eigenvalue analysis is repeated, and the two relevant modal branches are identified and tracked as their natural frequencies shift due to the blade-tip modification. The corresponding mode shapes are mass-normalized [Meirovitch 1986, He 2001], and the aerodynamic nodal-force vector is then projected onto this candidate-specific modal basis. The resulting generalized modal forces constitute the Modal Excitation Spectrum used in the present optimization.

The third computational domain is the optimization domain. The objective function is evaluated from the dominant MES value among the selected structural modal branches. Independent design variables and engineering constraints define the admissible candidate configurations. The optimizer generates new candidate designs, which are returned to the structural and modal evaluation until convergence. In contrast to conventional response-based procedures [Castanier 2006, Petrov 2010], the harmonic response solution is excluded from the optimization loop.

After convergence, the optimized design is transferred to the verification stage, where a conventional harmonic response analysis is used to evaluate displacement, vibration amplitude and dynamic stress. The individual mathematical operations are described in the following subsections.

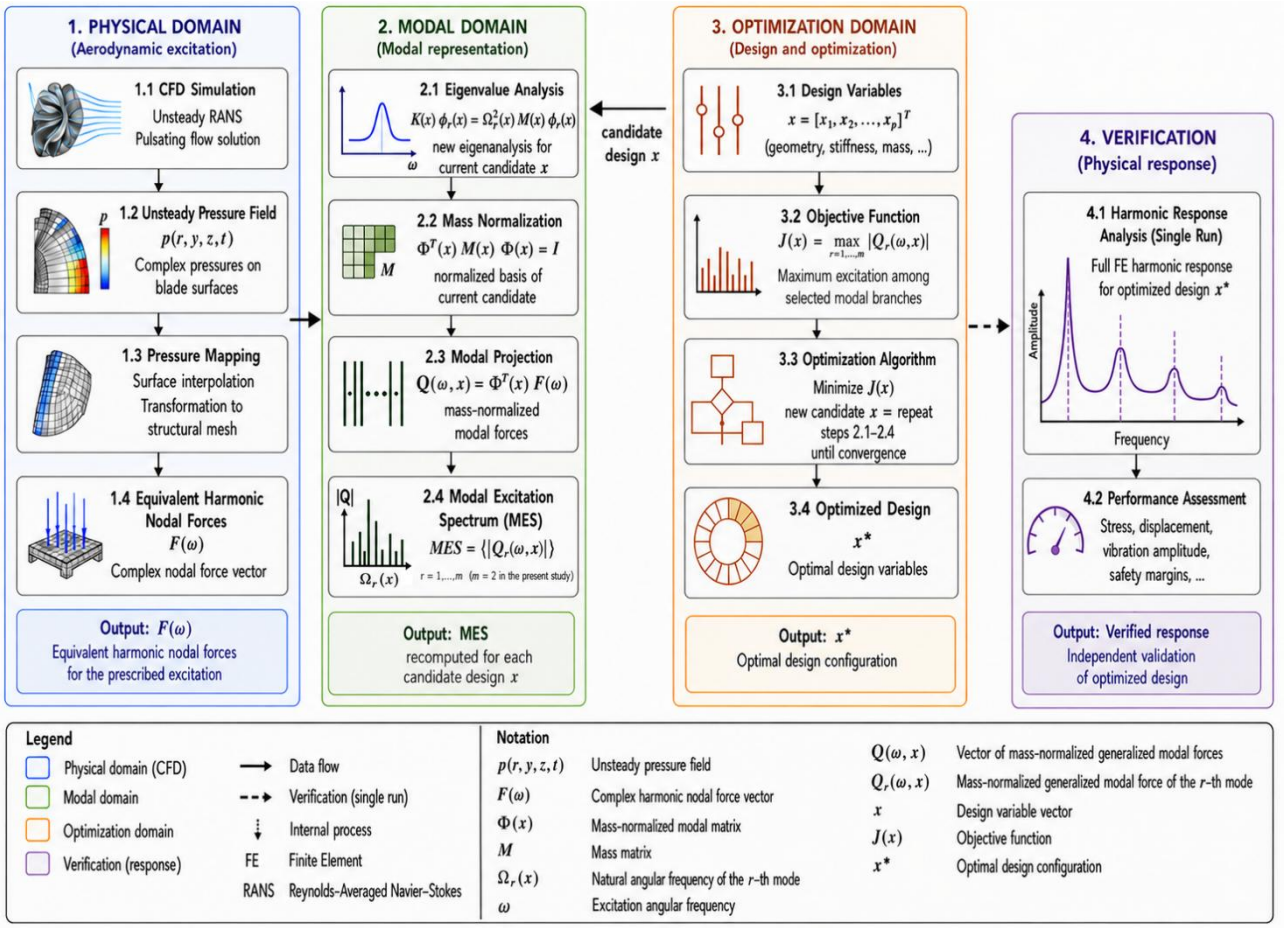


Figure 2. Overview of the proposed MES-based optimization framework. The structural model, modal basis and MES are recomputed for each candidate design, whereas a conventional harmonic response analysis is performed only after convergence for independent verification

3.2 Modal Representation of Aerodynamic Excitation

The computational workflow introduced in the previous section establishes the transformation of the optimization problem from the physical domain into the modal domain. The present section describes this transformation mathematically by converting the aerodynamic excitation acting on the structure into a modal representation suitable for subsequent optimization.

The aerodynamic excitation obtained from computational fluid dynamics is first transformed into an equivalent vector of harmonic nodal forces acting on the structural finite element model [Pan 2021, Hassan 2019]. This transformation preserves both the spatial distribution and the frequency content of the aerodynamic loading while expressing the excitation in a form directly compatible with the structural model. The equivalent harmonic loading is represented by the complex nodal force vector $\boldsymbol{F}(\boldsymbol{\omega})$, where ω denotes the excitation angular frequency. The corresponding time-domain representation of the harmonic excitation is

$$\boldsymbol{F}(t) = Re(\boldsymbol{F}(\omega)e^{i\omega t}) \tag{4}$$

where *Re* denotes the real part of the complex quantity and *i* is the imaginary unit.

Although the nodal force vector provides a complete description of the aerodynamic excitation in physical coordinates, it does not directly quantify the excitation of the individual structural vibration modes. For optimization purposes, the excitation is therefore projected onto the modal basis associated with the modal transformation defined in Eq. (2). Because the amplitudes of the eigenvectors obtained from the eigenvalue problem are arbitrary, the modal basis is mass-normalized [Bathe 1996, He 2001]. For real structural mode shapes, the normalized modal matrix $\boldsymbol{\Phi}$ satisfies

$$\boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{\Phi} = \boldsymbol{I} \tag{5}$$

where $\boldsymbol{M}$ is the global mass matrix, $\boldsymbol{\Phi}^T$ denotes the transpose of the normalized modal matrix and $\boldsymbol{I}$ is the identity matrix.

Mass normalization removes the arbitrary scaling of the eigenvectors and makes the resulting modal excitation levels comparable among different modes and candidate designs. This property is essential because modifications of geometry, stiffness or mass distribution alter both the natural frequencies and the corresponding mode shapes.

Projection of the aerodynamic excitation onto the normalized modal basis yields the vector of mass-normalized generalized modal forces [He and Fu 2001]

$$\boldsymbol{Q}(\omega) = \boldsymbol{\Phi}^T\boldsymbol{F}(\omega) \tag{6}$$

For a given harmonic excitation, the modal excitation is represented by the vector

$$\boldsymbol{Q}(\omega) = [Q_1(\omega), Q_2(\omega), \ldots, Q_r(\omega), \ldots, Q_m(\omega)]^{\mathrm{T}}, \tag{7}$$

where m denotes the number of structural modal branches included in the MES evaluation and $Q_r(\omega)$ is the complex generalized modal force associated with the r-th modal branch. In the present application, two relevant modal branches were considered, i.e. $m = 2$.

For each candidate design, the two relevant modal branches are identified and tracked as their natural frequencies shift. Their mode shapes are mass-normalized, and the harmonic aerodynamic loading is projected onto the updated modal basis. The resulting generalized modal forces constitute the Modal

Excitation Spectrum used in the optimization. The investigated operating-frequency interval serves to select and track the relevant modal branches rather than to define a frequency discretization of the MES.

The Modal Excitation Spectrum directly characterizes the excitation of the selected structural modal branches by the prescribed aerodynamic loading. Together with the tracked natural frequencies, it provides the modal quantities required for the constrained optimization formulated in the following section.

3.3 *Optimization in the Modal Excitation Domain*

The proposed framework uses the Modal Excitation Spectrum directly to reformulate the vibration-oriented design problem from the structural response domain into the modal excitation domain.

The objective is to reduce the strongest modal excitation among the selected structural modal branches. Because the optimum may correspond either to a uniform tuning change or to a non-uniform mistuning pattern, no particular circumferential distribution is prescribed in the general formulation.

Conventional optimization procedures usually perform a harmonic response analysis for every candidate design [Castanier 2006, Petrov 2010]. Although this approach directly predicts vibration amplitudes and dynamic stresses, repeated forced-response solutions generally represent the dominant computational cost.

In the proposed methodology, the optimization is performed directly on the MES. The design variables are collected in the vector

$$\boldsymbol{x} = \left[x_1, x_2, \dots, x_p\right]^T \tag{8}$$

where the individual parameters may represent blade geometry, local stiffness, mass distribution, material properties or other quantities influencing the structural dynamic behaviour. The components of $\boldsymbol{x}$ may be independent for the individual blades or shared among selected groups of blades.

The constrained optimization problem is formulated as

$$minimize\ J(\boldsymbol{x}) = \max|Q_r(\omega, \boldsymbol{x})| \tag{9}$$

$r = 1, 2, \dots, m$

subject to $\boldsymbol{x}_{min} \le \boldsymbol{x} \le \boldsymbol{x}_{max}$, $g_j(\boldsymbol{x}) \le 0$, $h_k(\boldsymbol{x}) = 0$,

where $Q_r(\omega,\boldsymbol{x})$ denotes the mass-normalized generalized modal force associated with the r-th selected modal branch and design vector $\boldsymbol{x}$. In the present application, the maximum was evaluated over the two tracked modal branches. The functions g_j and h_k represent inequality and equality constraints, respectively. They may describe geometric bounds, manufacturing requirements, allowable mass changes or rotor-balance conditions. The maximum MES value is selected because the objective is to suppress the most strongly excited mode among the modal branches included in the evaluation.

For each candidate design, the objective function is determined from the maximum amplitude of the generalized modal force over the investigated excitation-frequency range and the two tracked modal branches. The frequency range corresponds to the specified rotor-speed interval for the excitation order EO = 24. Thus, the objective function represents the strongest generalized modal excitation identified within the considered operating range, rather than the excitation evaluated at a single prescribed frequency.

The objective function is evaluated without performing a harmonic response analysis during each optimization iteration. For every candidate design x, the structural finite element model is updated, a new eigenvalue analysis is performed, the relevant mode shapes are mass-normalized and the MES is recomputed using the current modal basis. The maximum generalized modal force is therefore used as an excitation-based optimization indicator rather than as a direct measure of vibration amplitude or dynamic stress. The forced response also depends on the frequency separation between excitation and the corresponding natural frequency, modal damping and modal dynamic compliance. Consequently, a reduction in the MES-based objective does not generally imply a proportional reduction in forced response. For this reason, the optimized design is independently assessed by a conventional harmonic-response analysis over the complete investigated operating range.

The formulation is independent of the selected optimization algorithm. Gradient-based methods, evolutionary algorithms and other numerical strategies may therefore be employed, provided that they evaluate the MES-based objective function and satisfy the prescribed constraints.

The framework thus provides a computationally efficient optimization strategy that retains the physical relationship between aerodynamic excitation and structural dynamics while allowing the optimum design to be tuned, intentionally mistuned or partially constrained by practical engineering requirements.

4 APPLICATION TO A TURBOCHARGER TURBINE WHEEL

The proposed Modal Excitation Spectrum-based optimization framework was applied to the turbine wheel of a turbocharger intended for a large marine engine. The investigated wheel consisted of thirteen rotor blades, whereas the stationary nozzle ring contained twenty-four stator vanes. The resulting rotor–stator interaction therefore generated a dominant excitation order EO = 24. The corresponding excitation frequency was related to the rotor speed n by $f = 24\ n/60$ and reached 15,840 Hz at the nominal speed of 39,600 rpm.

The investigated excitation-frequency interval extended from 15,200 to 16,600 Hz, which for EO = 24 corresponded to a rotor-speed range of 38,000–41,500 rpm. This interval included the nominal turbocharger speed and contained a near-resonant condition that could not be avoided by changing the operating requirements. The maximum equivalent stress over the harmonic phase cycle consequently reached an elevated level in the blade, indicating a potentially critical dynamic response and motivating further optimization of the structural dynamic properties of the wheel.

The aerodynamic excitation was determined by transient CFD analysis and transferred to the structural finite element model in the form of complex harmonic nodal forces. Following the procedure described in Section 3, the forces were projected onto the mass-normalized modal basis and evaluated using the Modal Excitation Spectrum. During optimization, the aerodynamic excitation obtained for the original geometry was retained, whereas the structural modal properties and the MES were updated for each candidate design. This approximation was adopted because the admissible material removal was small and confined to the blade-tip region. After convergence, an additional CFD analysis was performed for the optimized blade geometry. The resulting pressure distribution and equivalent harmonic nodal forces differed only negligibly from those obtained for the original geometry. The updated nodal forces were subsequently used in the final verification harmonic-response analysis with a phase sweep.

The total damping used in the final harmonic-response analyses comprised material and aerodynamic contributions. The material damping ratio was set to ζ_m = 0.0001 (0.01%), while the aerodynamic damping ratio, ζ_{Aero} = 0.0014 (0.14%), was

calculated using CFD and included as an aerodynamic contribution to the total damping ratio of the dynamic model. The same damping values were applied to both the original and optimized turbine wheel configurations to ensure a consistent comparison of their dynamic responses.

Thirteen independent design variables were introduced, each defining the geometric modification of the tip of one blade. The adopted parameterization therefore permitted different modifications of the individual blades and, consequently, the introduction of an intentional blade-to-blade variation. Because the thirteen-bladed wheel did not permit simple diametrically paired modifications, preservation of dynamic balance was imposed as an explicit optimization constraint. The balance condition was imposed independently of the individual design variables and did not require their values to be identical.

The blade-tip modification was represented by a discrete sequence of progressively enlarged material-removal regions extending from the blade tip along both bounding edges of the blade profile. For each blade, the admissible modification ranged from the unmodified geometry, corresponding to zero material removal, to a maximum removal of 3% of the original blade mass. The optimized design corresponded to the removal of 2.12 g per blade, representing approximately 1.06% of the original blade mass. Although the thirteen blade modifications were treated as independent design variables, the balance-constrained optimization converged to the same modification for all blades.

The optimization was performed using the Multi-Objective Genetic Algorithm (MOGA). A maximum of 500 candidate-design evaluations was prescribed. Convergence was identified from the stabilization of the best objective-function value, and the optimization terminated after approximately 240 evaluations. Two relevant structural modal branches were included in the MES evaluation and tracked throughout the optimization as their natural frequencies shifted. These modes were selected because their natural frequencies lay within the investigated excitation-frequency interval.

The stability of the optimization result with respect to the discretization of the design variables was assessed by an additional optimization run using a finer discretization. Although the refined discretization increased the number of candidate-design evaluations, the resulting optimal configuration and the corresponding objective-function value remained practically unchanged. This result supports the adequacy of the original discretization and indicates a low sensitivity of the identified optimum to the selected discretization step.

5 RESULTS AND DISCUSSION

For the original turbine wheel, the maximum equivalent stress over the harmonic phase cycle was evaluated throughout the investigated operating-speed interval. Its highest value is shown in Figure 3 and occurred at an excitation frequency of 16,428 Hz, corresponding to a rotor speed of 41,070 rpm for *EO* = 24. The predicted stress level indicated a potentially critical dynamic response and raised concerns regarding fatigue durability under the required operating conditions. This result motivated the optimization of the structural dynamic properties of the wheel.

The optimization converged to a solution in which the same geometric modification was obtained for the tips of all thirteen blades. Importantly, this uniform modification was not prescribed in advance. It emerged from the optimization involving independent blade variables and the explicit dynamic-balance constraint.

For the optimized turbine wheel, the maximum equivalent stress over the harmonic phase cycle was evaluated throughout the same operating-speed interval. Its highest value occurred at an excitation frequency of 15,368 Hz, corresponding to a rotor speed of 38,420 rpm for *EO* = 24. The result obtained from the verification harmonic response analysis is shown in Figure 4.

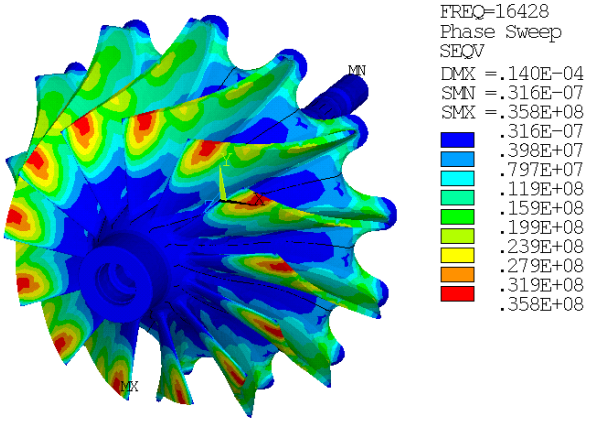


Figure 3. Maximum equivalent stress over the harmonic phase cycle in the original turbine wheel at the governing response within the investigated operating-speed interval (*f* = 16,428 Hz; *n* = 41,070 rpm for *EO* = 24).

The maximum equivalent stress over the harmonic phase cycle, subsequently maximized over the investigated operating-speed interval, decreased from 35.8 MPa in the original configuration to 1.20 MPa in the optimized configuration, corresponding to a reduction of 96.6%. The two governing values occurred at different excitation frequencies because the modification changed the structural modal properties of the wheel. The verification analysis employed aerodynamic loading obtained from the additional CFD calculation of the optimized geometry. Because the resulting pressure distribution and equivalent harmonic nodal forces differed only negligibly from those of the original configuration, the substantial stress reduction can be attributed predominantly to the change in structural modal properties rather than to a change in aerodynamic excitation.

The negligible change in aerodynamic loading obtained for the final design supports the assumption adopted during optimization for the present relatively small and localized blade-tip modification. This result should not be generalized to larger geometric changes, for which the unsteady aerodynamic pressure field may need to be recomputed during the optimization process.

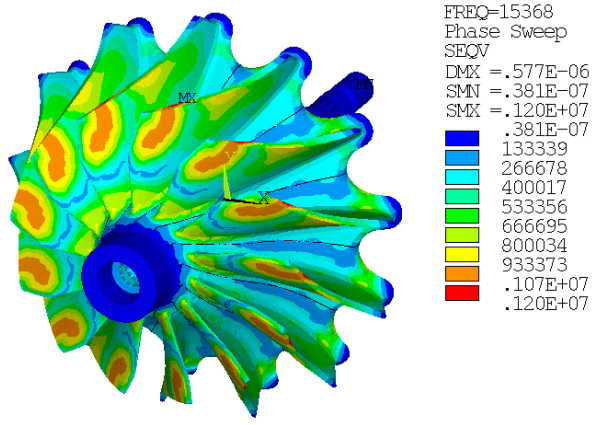


Figure 4. Maximum equivalent stress over the harmonic phase cycle in the optimized turbine wheel at the governing response within the same operating-speed interval (*f* = 15,368 Hz; *n* = 38,420 rpm for *EO* = 24). The colour scale is adjusted to the substantially lower stress level of the optimized configuration.

Although the parameterization permitted non-uniform blade-tip modifications, the optimum obtained for the investigated

excitation and operating range retained the cyclic symmetry of the wheel. Consequently, no additional circumferential mass asymmetry was introduced, and the dynamic balance requirement was satisfied. This result also demonstrates that the proposed framework does not impose intentional mistuning as a necessary outcome but identifies the most favourable admissible distribution of the blade modifications for the specific operating conditions and design constraints.

The successful verification of the optimized design by the subsequent harmonic response analysis supports the use of the Modal Excitation Spectrum as a suitable indicator for identifying beneficial structural modifications. Candidate designs could be evaluated primarily in the modal excitation domain, whereas the computationally more demanding harmonic response analysis was required only for the final verification.

The optimization required approximately 240 candidate-design evaluations. A conventional response-based optimization would therefore have required approximately 240 complete harmonic-response analyses. In the proposed workflow, harmonic-response analysis was excluded from the optimization loop and performed only for final verification. The number of harmonic-response analyses required during optimization and verification was thus reduced from approximately 240 to one, corresponding to a reduction of approximately 99.6%. Each candidate evaluation nevertheless required updating the structural model, performing an eigenvalue analysis and recomputing the generalized modal forces. The reported saving therefore concerns the elimination of repeated harmonic-response analyses rather than an equivalent reduction in total computational time.

For the investigated turbine wheel, the reduction in the MES-based objective was accompanied by a substantial decrease in maximum equivalent stress in the independent harmonic-response analysis. This agreement supports the suitability of the selected indicator for the present application but should not be interpreted as a general equivalence between modal excitation and forced-response amplitude.

The case study demonstrates that the proposed methodology can incorporate independent blade modifications together with practical rotor-balance and manufacturing requirements. A substantial reduction in the maximum equivalent stress over the harmonic phase cycle was achieved without changing the required turbocharger operating conditions and with only a relatively small modification of the blade-tip geometry.

6 CONCLUSIONS

A Modal Excitation Spectrum-based framework for the optimization of aerodynamically excited bladed structures was presented. The method evaluates the interaction between the aerodynamic excitation and the structural modes of each candidate design and enables potentially critical configurations to be assessed without performing a complete harmonic response analysis during every optimization step.

The framework was applied to a thirteen-bladed turbocharger turbine wheel operating in a speed range containing a near-resonant condition. The original design exhibited an elevated maximum equivalent stress over the harmonic phase cycle, indicating a potentially critical dynamic response under the investigated operating conditions.

Thirteen independent blade-tip design variables were considered together with an explicit dynamic-balance constraint.

The optimization converged to a uniform and readily manufacturable modification of all blade tips, despite permitting different modifications of the individual blades. The resulting configuration retained the cyclic symmetry and dynamic balance of the wheel. An independent harmonic response analysis confirmed that the maximum equivalent stress over the harmonic phase cycle, subsequently maximized over the investigated operating-speed interval, was reduced by 96.6%.

The results demonstrate that the proposed framework can efficiently identify structural modifications that suppress critical dynamic responses while respecting practical design, manufacturing and rotor-balance constraints. Further work will investigate operating conditions for which balance-constrained non-uniform blade modifications may provide an additional reduction in vibration response, together with experimental verification of the optimized designs.

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