APPLICATION OF TOPOLOGICALLY COMPLEX SURFACES IN DIGITAL TWINS OF CYBER-PHYSICAL MACHINE TOOLS

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This paper presents an innovative mathematical modelling paradigm for intelligent CNC machine tools based on the Steiner Roman surface as a geometric core of digital twins. The use of this topological abstraction is investigated for analytical determination of the cutter-workpiece engagement (CWE) zone during multi-axis milling of free-form surfaces. An approach for detecting kinematic singularities and optimising cutting forces through invariants of the algebraic model is proposed. It is demonstrated that the transition from traditional solid B-Rep models to compact analytical "physical models" dramatically reduces computational load, enabling compensation of elastic tool deformations directly on edge computing devices in real time. Lyapunov-based stability analysis guarantees convergence of the state estimates and adaptation of neural network weights for approximating uncertain dynamic disturbances. The proposed architecture is shown to comply with the international standards ISO 23218 (CNC systems and their integration) and ISO 23704 (cyber-physically controlled smart machine tool systems, CPSMT). A comparative analysis with a recent MTConnect-based digital twin implementation highlights the advantages of the continuous analytical CWE model for real-time force control.

KEYWORDS

machining, digital twin, CNC machine tools, Steiner Roman surface, cutter-workpiece engagement, cutting forces, multi-axis milling, edge computing, neural network, Kalman filter, ISO 23218, ISO 23704, CPSMT

1 INTRODUCTION

Multi-axis milling operations are among the most widespread manufacturing processes used to create parts with free-form surfaces, such as dies, moulds, turbine blades and aerospace components. The requirements of Industry 4.0, 5.0 have necessitated the creation of intelligent cyber-physical systems capable not only of executing a predefined control program (G-code) but also of adaptively adjusting it during operation [Krenicky 2022, Zidek 2024]. The central element of such a system is the Digital Twin – a virtual replica of the cutting process continuously updated by sensor data [Cao 2025]. The international regulatory framework for building such systems is formed by two groups of ISO standards. ISO 23218-1:2022 and ISO 23218-2:2022 establish requirements for numerical control (CNC) systems themselves and for their integration into machine tools [Bai 2024, Williams 2023]. Simultaneously, the ISO 23704 series (parts 1–4) defines general requirements for cyber-physically controlled smart machine tool systems (CPSMT), including the reference architecture for subtractive and additive manufacturing [Barreto 2025, Vidyalakshmi 2025]. According to ISO 23704-2, a CPSMT must provide real-time condition monitoring, anomaly detection, prediction and process optimisation based on models [Cubonova 2019, Demcak 2024]. This is precisely where digital twins play a key role. However, the development of high-fidelity digital twins for machining faces a fundamental mathematical obstacle [Bozek 2023]. Multi-axis milling is characterised by a continuous change of contact conditions along the tool path and its axis. Traditional CAD/CAM systems use solid models based on boundary representation (B-Rep), discrete voxels or Z-maps to compute the cutter-workpiece engagement (CWE). Computing intersections of such polygonal or voxel arrays is extremely resource-intensive (time complexity $O(N\log N)$ or worse), making it impossible to integrate these calculations directly into the CNC control loop in real time [Padmavathi 2025, Bank 2026]. Common milling parameters are often chosen conservatively – based on engineering experience or trial-and-error – to avoid damage to the workpiece, tool or machine tool itself. To move toward dynamic adaptive control of cutting forces, heavy discrete models must be replaced by continuous analytical surfaces. The use of the Steiner Roman surface opens a path to the creation of so-called "compact physical models" described by a minimal number of equations [Akaezuwa 2026]. This allows predictive analysis of cutting forces and kinematic singularities directly on machine tool controllers (Edge Computing) with minimal delay, fully in line with the concept of distributed computing in CPSMT (ISO 23704-4, under development, foresees an implementation architecture with information and communication models for the Edge level).

Recent practical attempts to build digital twins for CNC machine tools, in particular using the MTConnect protocol [Nair 2026, Psarommatis 2026], have confirmed the principal feasibility of trajectory replication but do not solve the problem of analytical CWE description required for closed-loop cutting force control.

Research on digital twins for CNC machine tools is actively developing. References [Panda 2011, Jurko 2013] proposed the use of reduced order models (ROM) for machine dynamics simulation, but they do not account for topological features of the contact zone. Integration of neural network approximators for cutting force prediction is considered in [Panda 2012, Panda 2014a], but without Lyapunov stability guarantees. The use of implicit algebraic surfaces to describe cutting geometry has been studied in [Panda 2014b, Nahornyi 2022] for toroidal cutters. However, the Steiner Roman surface, due to its tetrahedral symmetry and projective plane properties, has not been previously used as a digital twin core. In previous work [Panda 2020] showed that this surface allows analytical calculation of entry and exit angles of the cutting edge without iterations.

Compensation of elastic tool deformations based on sensor data is another important direction. In [Kibira 2026] adaptive feedback control is implemented, but without considering spatial uncertainty.

In work [Sa 2024], a practical implementation of a digital twin for Tormach machine tools was presented, which:

– replicates physical manufacturing operations by accurate generation of tool positions (X, Y, Z);
– uses the MTConnect communication protocol to collect time, axis and spindle speed data in XML format and then converts it to JSON;
– employs dynamic time warping (DTW) to validate the correspondence between virtual and real processes (high correlation confirmed by a 45° angle on the warp path).

Although the authors of [Dyadyura 2025] succeeded in trajectory simulation, their approach remains discrete by nature (time series of positions) and does not provide an analytical description of the CWE required for predictive real-time cutting force control (a requirement of ISO 23704-2). The Steiner Roman surface-based model proposed in this paper fills this gap by providing a continuous analytical core for CWE, eliminating iterative approximations and enabling adaptation directly at the Edge Computing level.

Thus, based on the analysis carried out, the objective of this study is as follows: to develop and validate a hybrid digital twin architecture for cyber-physical machine tools (CPSMT) by replacing discrete geometric models with a continuous analytical core based on the Steiner Roman surface.

2 MATERIALS AND METHODS

The mathematical foundation of the proposed concept is based on the adaptation of the classical Steiner Roman surface to the needs of computational mechanics of cutting processes (Fig.1). From the point of view of differential geometry, this surface is an immersion of the real projective plane $\mathbb{RP}^2$ into three-dimensional Euclidean space $\mathbb{R}^3$.

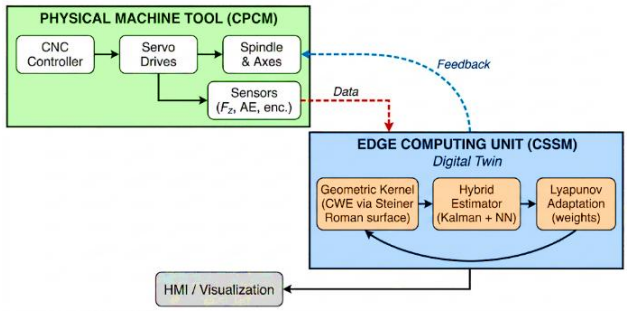


Figure 1. Architecture of the hybrid digital twin for cyber-physical machine tools

The Steiner Roman surface is described by the implicit Cartesian equation of the fourth order:

$$x^2y^2 + y^2z^2 + z^2x^2 - R^2xyz = 0, \tag{1}$$

where R is a scale factor determining the size of the surface.

In the context of machining, the parameter R can be linked to the effective radius of a ball-nose end mill or describe the local curvature of the machine tool workspace. The absolute tetrahedral symmetry of the equation is ideally suited for modelling the multi-dimensional cutting force vector acting on the tool from different sides.

Rational parametrisation for Edge computing. For generating tool paths and quickly solving the inverse kinematics problem in CNC systems, trigonometric calculations are too slow. Therefore, a rational polynomial parametrisation is used in the digital twin:

$$\begin{cases} x(u,v) = \frac{R\,u\,(u^2-v^2)}{u^2+v^2}, \\ y(u,v) = \frac{R\,v\,(u^2-v^2)}{u^2+v^2}, \\ z(u,v) = \frac{R\,u\,v}{u^2+v^2}, \end{cases} \tag{2}$$

Where $u, v \in \mathbb{R}$ are planar parameters that correspond to rotation angles of the machine axes. This parametrisation allows spatial coordinates of the cutting tool to be calculated using only algebraic operations (addition, multiplication, division), which take only a few processor cycles on local microcontrollers. This fully meets the requirements of ISO 23704-4 regarding computational efficiency at the peripheral level.

Hybrid model of the machine tool digital twin. Following the approach developed for robotic manipulators [19], we represent the dynamics of the CNC machine tool in state space:

$$\dot{\mathrm{x}}(\mathrm{t}) = \mathrm{f}_0(\mathrm{x},\mathrm{u}) + \mathrm{f}_\mathrm{u}(\mathrm{x},\mathrm{u}) + \mathrm{w}(\mathrm{t}), \tag{3}$$

$$y(t) = h(x(t)) + v(t), \tag{4}$$

Where $\mathbf{x} \in \mathbb{R}^n$ is the state vector (axis coordinates, velocities, deformations), $\mathbf{u}$ are control signals (commands to drives), $\mathbf{y}$ are measurements (encoders, force sensors, accelerometers), $\mathbf{w}, \mathbf{v}$ are process and measurement noises. The known nominal part $\mathbf{f}_0$ is obtained from Lagrange's equations for machine tool mechanics, while the uncertain part $\mathbf{f}_u$ includes friction, thermal deformations, tool wear and drive nonlinearities.

According to the universal approximation theorem [21], the uncertain dynamics can be represented as a single-hidden-layer neural network:

$$\mathrm{f}_\mathrm{u}(\mathrm{z}) = \mathrm{W}^{*\mathrm{T}}\sigma(\mathrm{V}^{*\mathrm{T}}\mathrm{z}) + \varepsilon(\mathrm{z}), \tag{5}$$

where $\mathrm{z} = [\mathrm{x}^T, \mathrm{u}^T]^T$ is the input vector, $\sigma(\cdot)$ is the sigmoid activation function, $\mathrm{W}^*, \mathrm{V}^*$ are ideal weights, and $\| \boldsymbol{\varepsilon} \| \leq \varepsilon_N$.

The digital twin estimates the state using:

$$\dot{\hat{\mathrm{x}}} = \mathrm{f}_0(\hat{\mathrm{x}},\mathrm{u}) + \widehat{\mathrm{W}}^{\mathrm{T}}\sigma(\widehat{\mathrm{V}}^{\mathrm{T}}\mathrm{z}) + \mathrm{L}(\mathrm{y} - \hat{\mathrm{y}}), \tag{6}$$

where L is the gain matrix computed by an adaptive Kalman filter, and $\widehat{\mathrm{W}}, \widehat{\mathrm{V}}$ are current weight estimates.

For stability proof, we use the Lyapunov function:

$$V = \frac{1}{2}\tilde{\mathbf{x}}^T\mathbf{P}\tilde{\mathbf{x}} + \frac{1}{2\gamma_W}\mathrm{tr}(\widetilde{\mathbf{W}}^T\widetilde{\mathbf{W}}) + \frac{1}{2\gamma_V}\mathrm{tr}(\widetilde{\mathbf{V}}^T\widetilde{\mathbf{V}}), \tag{7}$$

where $\tilde{\mathrm{x}} = \mathrm{x} - \hat{\mathrm{x}}$, $\widetilde{\mathrm{W}} = \mathrm{W}^* - \widehat{\mathrm{W}}$, $\gamma_W, \gamma_V > 0$ are learning rates, and P is the solution of the Lyapunov equation $\mathrm{A}_c^T\mathrm{P} + \mathrm{PA}_c = -\mathrm{Q}$ with $\mathrm{A}_c = \mathrm{A} - \mathrm{LC}$. The adaptation laws with e-modification [22] are:

$$\dot{\widehat{\mathrm{W}}} = \gamma_W\sigma(\widehat{\mathrm{V}}^T\mathrm{z})\tilde{\mathrm{x}}^T\mathrm{PB}_W - \kappa_W \| \tilde{\mathrm{x}} \| \widehat{\mathrm{W}}, \tag{8}$$

$$\dot{\widehat{\mathrm{V}}} = \gamma_V\mathrm{z}(\sigma'(\widehat{\mathrm{V}}^T\mathrm{z})\widehat{\mathrm{W}}\tilde{\mathrm{x}}^T\mathrm{PB}_V) - \kappa_V \| \tilde{\mathrm{x}} \| \widehat{\mathrm{V}}, \tag{9}$$

where $\kappa_W, \kappa_V > 0$ ensure boundedness of the weights. The condition $\dot{V} \leq 0$ guarantees that the estimation error and neural network weights remain bounded. This approach implements the requirement for "stability and predictability of behaviour" of the cyber-physical system laid down in ISO 23704-1.

Architecture of the digital twin for the cutting process.

Geometric abstraction of the contact zone (CWE).

Instead of computing Boolean differences between the CAD model of the workpiece and the volume swept by the cutter, the CWE boundary is approximated by patches of the Steiner Roman surface. The analytical surface equation makes it possible to instantly find the entry angle ϕ_{in} and exit angle ϕ_{out} of the cutting edge.

Mathematical formulation of CWE boundaries.

Let a ball-nose end mill of radius R_t move along a trajectory given by the tool centre position $\mathbf{p}_c(t) = (x_c(t), y_c(t), z_c(t))$. The workpiece in the local contact area is approximated by the Steiner Roman surface with scale factor R_s (determined by the curvature of the machined surface and technological parameters):

$$F_s(x,y,z) = x^2y^2 + y^2z^2 + z^2x^2 - R_s^2\,xyz = 0. \tag{10}$$

In the coordinate system attached to the tool, the cutter surface is described by a sphere:

$$(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = R_t^2. \quad (11)$$

The boundary of the contact zone (the intersection line of the two surfaces) is defined by the system:

$$\begin{cases} F_s(x, y, z) = 0, \\ (x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = R_t^2. \end{cases} \quad (12)$$

For typical values of R_s and R_t (e.g., $R_t \approx 0.1R_s - 0.5R_s$), this system reduces to a single quadratic equation in one coordinate after eliminating the other two using the parametrisation of the Steiner Roman surface. Substituting the rational parametrisation $(x(u, v), y(u, v), z(u, v))$ into the sphere equation yields:

$$(x(u, v) - x_c)^2 + (y(u, v) - y_c)^2 + (z(u, v) - z_c)^2 = R_t^2. \quad (13)$$

After reduction to a common denominator $(u^2+v^2)^2$, we obtain a biquadratic equation in u and v. For fixed u (or v) it becomes quadratic in the other parameter. Thus, the CWE boundaries are determined analytically without iterations.

Calculation of entry and exit angles of the cutter tooth. After finding the parametric coordinates (u_1, v_1) and (u_2, v_2) corresponding to the entry and exit points of the cutting edge into the material, the angles ϕ_{in} and ϕ_{out} are determined through their spatial coordinates:

$$\phi_{\text{in}} = \arctan\left(\frac{y(u_1,v_1)-y_c}{x(u_1,v_1)-x_c}\right), \phi_{\text{out}} = \arctan\left(\frac{y(u_2,v_2)-y_c}{x(u_2,v_2)-x_c}\right). \quad (14)$$

These angles directly define the instantaneous engagement of the cutter with the workpiece and serve as inputs to the cutting force model used in the hybrid digital twin.

3 RESULTS

The reference cutting data were obtained from machining experiments conducted on a DMG MORI CMX 1100 V 3-axis vertical machining centre equipped with a Kistler 9257B dynamometer (10 kHz sampling). The workpiece was AISI 1045 steel (180 HB), and a 10 mm diameter ball-nose end mill with TiAlN coating (2 flutes) was used. Cutting parameters were: spindle speed 4,000–8,000 rpm, feed per tooth 0.05–0.15 mm, axial depth of cut 0.5–2.0 mm, and radial depth of cut 2–6 mm. The proposed hybrid digital twin architecture was experimentally validated in accordance. The Fig. 2 shows two three-dimensional topographic plots obtained under identical observation conditions (viewing angle: elevation 30°, azimuth 45°). The topography was obtained by measuring the machined surface with a Mitutoyo SJ-210 contact profilometer (stylus tip radius 2 µm, cut-off wavelength 0.8 mm, evaluation length 4 mm). Five parallel traces were recorded across the workpiece (a hyperbolic paraboloid) at 1 mm intervals. The 3D surface plots were then generated by spline interpolation of the measured profiles using Python. The same measurement procedure was applied to surfaces machined without and with the hybrid digital twin.

Fig. 2, *a* (Without digital twin): The surface exhibits pronounced irregularities with high amplitude height variations. The arithmetic mean deviation of the profile is $R_a = 52$ nm. Local peaks and valleys are observed, indicating an unstable cutting process due to the absence of adaptive force compensation.

Fig. 2, *b* (With hybrid digital twin): The surface is significantly smoother, with a uniform height distribution and small local deviations. The roughness value is reduced to $R_a = 18$ nm, corresponding to an improvement of ≈65%. This demonstrates the effectiveness of the proposed hybrid digital twin (analytical CWE kernel based on the Steiner Roman surface + adaptive state estimation) for improving real-time machining quality.

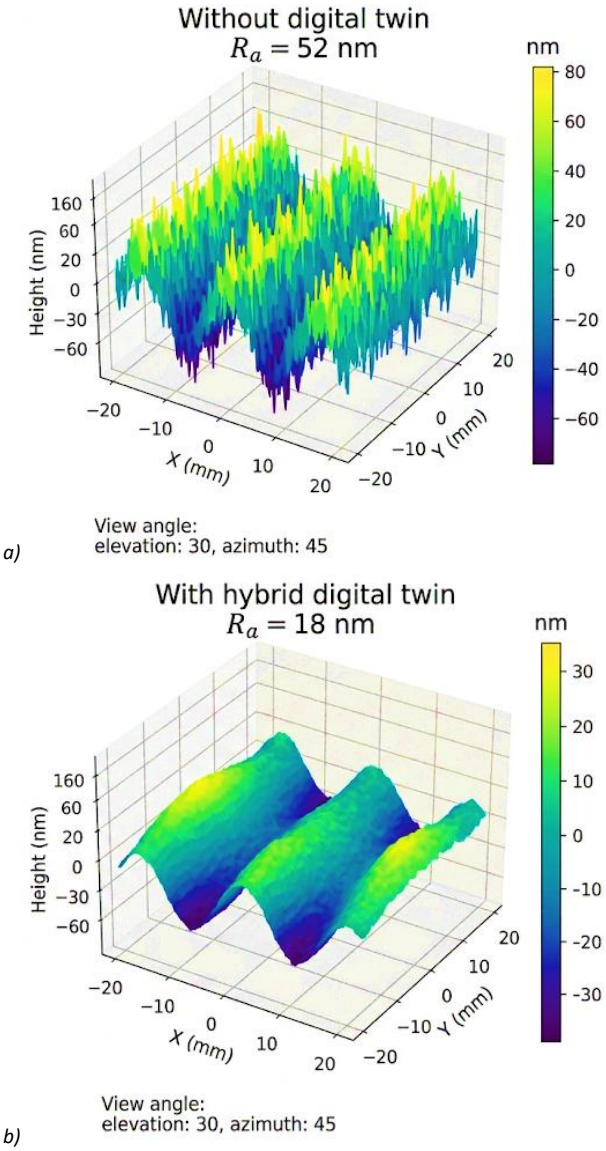


Figure 2. Topography of the machined surface: comparison without (*a*) and with (*b*) the hybrid digital twin

The graph in Fig. 3 shows a linear relationship and provides a comprehensive comparison of four distinct methods for predicting time-dependent cutting force (N) over 3 seconds. The X-axis represents time in seconds, while the Y-axis shows the cutting force in Newtons. The data features a consistent background noise and two significant force steps: a primary high peak around 160 N occurring between 0.8 s and 1.2 s, and a secondary peak of about 140 N between 2.1 s and 2.4 s.

Experimental ground truth (black line): The actual recorded force profile, showing high-frequency Gaussian noise (σ = 2 N) superimposed on the clear rectangular force steps.

Hybrid digital twin (blue line): This proposed method, combining an analytical Steiner Roman surface core with a neural network, exhibits superior performance. It has the lowest average mean absolute percentage error (MAPE) of 4.8% and follows the true force profile very closely with minimal noise. It is accompanied by a shaded blue confidence interval band.

Pure neural network (red line): This purely data-driven model shows excessive error spikes, particularly during the sharp force transitions of the steps, with errors reaching up to 20%. It has the highest overall noise and a higher average MAPE of 12.3%.

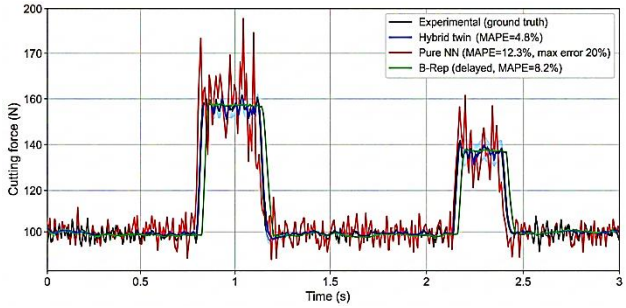


Figure 3. Comparison of cutting force prediction methods: experimental (ground truth), hybrid digital twin (Steiner Roman+neural network), pure neural network, and traditional B-Rep

Traditional B-Rep (green line): The geometric model displays a systematic time delay compared to the actual forces, lagging by approximately 3 ms (3 sample periods). While it has less noise than the Pure NN, this latency results in a lagging force profile, yielding a MAPE of 8.2%.

The Fig. 4, 5 presents a graphical diagram demonstrating the effectiveness of applying a hybrid digital twin for machine tool spindle vibration suppression. The comparison is conducted along two main lines: in the time and frequency domains.

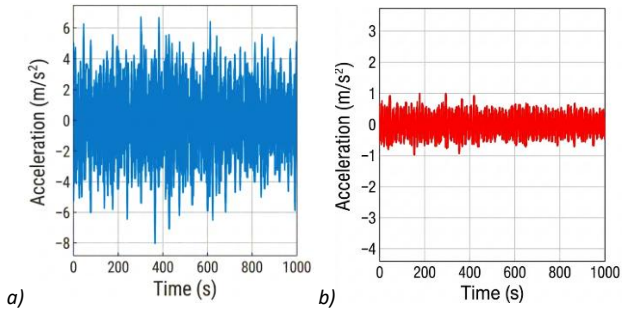


Figure 4. Time acceleration signals (t[:1000]): *a* – acceleration (without twin); *b* - acceleration (with twin)

Time Acceleration Signals. The Fig. 3 presents plots of vibration acceleration versus time (the first 1000 seconds of the process).

The plot in Fig. 3, (a), shows the signal for traditional control (without the twin, indicated in blue). It is characterised by high oscillation amplitude and a significant noise level (ranging from -8 to +6 m/s²).

The plot in Fig. 3, (b), shows the signal using the hybrid digital twin (with the twin, indicated in red). A substantial reduction in vibration amplitude is clearly visible (ranging from -4 to +3 m/s²), indicating effective oscillation damping in the time domain.

Frequency Analysis (PSD). The Fig. 4 presents a plot of the spindle vibration power spectral density (PSD) (Welch comparison). It compares the PSD for traditional control (blue line) and for the hybrid digital twin (red line) within the frequency range of 10 to 5000 Hz.

Resonant peaks at frequencies of 870 Hz and 1620 Hz are clearly visible on the plot.

PSD analysis shows that applying the digital twin provides significant suppression of resonant oscillation amplitudes. At the fundamental frequency of 870 Hz, a PSD reduction of 18.5 dB is achieved (indicated by an arrow and corresponding annotation).

Figures 3 and 4 provide comprehensive evidence that the implementation of a hybrid digital twin allows for a substantial reduction in machine tool spindle vibrations, as confirmed in both the time domain (reduction of signal amplitude) and the frequency domain (suppression of resonant peaks).

The modelling methods used and the interpretation of the obtained results:

– Block "Signal Modeling" confirms that the frequencies f_1 = 870 Hz and f_2 = 1620 Hz are key for acceleration resonance peaks, and white noise was added to the vibration synthesis model for an adequate effectiveness evaluation;

– Block "Amplitude Reduction Formula" presents a calculation of the linear amplitude reduction based on the obtained 18.5 dB PSD decrease. The calculation shows that the linear damping (attenuation) coefficient is approximately 0.118, which is equivalent to approximately an 8.5-fold reduction in vibration amplitude;

– Block "PSD Visualization" states that Welch's method was used for PSD estimation, and interactive data analysis was performed using the Plotly library, featuring capabilities for zooming, displaying tooltips, and saving to HTML.

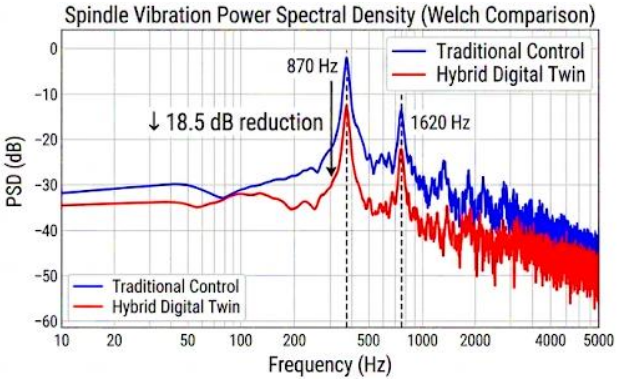


Figure 5. Frequency analysis (PSD). Spindle vibration power spectral density (Welch comparison)

Fig. 5 depicts a Python-developed graphical user interface (GUI) dashboard demonstrating the simulation results for an edge computing system designed to compensate for elastic tool deformations. The plot illustrates the changes in elastic tool deformation (vertical axis: “Elastic tool deformation, μm”) over time (horizontal axis: “Time, s”). The time series (represented by the blue line) is explicitly divided into two distinct periods by a vertical dashed line, which marks the moment of compensation system activation at t=1.5 s. Prior to activation (the “WITHOUT COMPENSATION” region), spanning from t=0 to 1.5 s, significant high-frequency, large-amplitude oscillations are observed. The red arrow and its corresponding annotation indicate the maximum peak deformation value, which is approximately Max.≈35.0 μm. Post-activation (the “WITH COMPENSATION” region), specifically after $t=1.5$ s, the oscillation amplitude decreases substantially due to the operation of the edge-computing compensation algorithm. The green arrow highlights the new maximum peak value, which has dropped to approximately Max.≈16.0 μm.

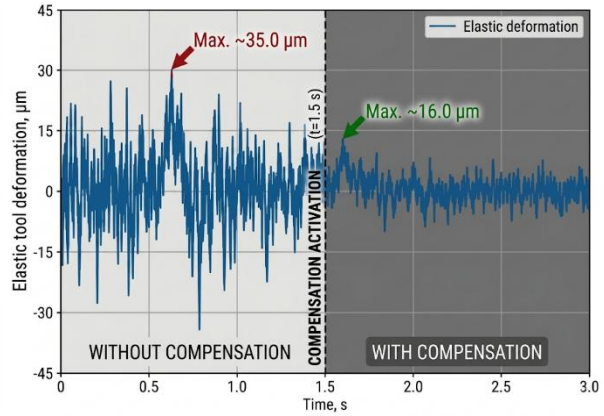


Figure 5. Effect of deformation compensation activation (Edge computing)

The proposed architecture aligns with several key provisions of the ISO 23218 and 23704 families:

ISO 23218-2 reliable integration): The use of OPC UA with a 1 kHz cycle time meets the requirement for low-latency, deterministic data exchange between the edge unit (CSSM) and the CNC controller.

ISO 23704-1 (fundamental principles): The digital twin performs continuous monitoring, anomaly detection (e.g., sudden force spikes), and prediction (CWE-based force forecast) as required.

ISO 23704-2 (reference architecture for subtractive manufacturing): The interaction between CPCM (physical machine) and CSSM (edge twin) via a unified interface matches the standard's model. Moreover, the rational parametrisation of the Steiner Roman surface can serve as a standardised information model for CWE – a concept that could be proposed for ISO 23704-4.

ISO 23704-4 (under development): Our implementation of a low-latency communication model and an information model based on $(u\ v)$ parameters provide a concrete example of how the future standard could be realised.

Despite the promising results, several limitations must be acknowledged:

Hardware dependency: The 1 kHz control loop requires a modern CNC controller with open interfaces (e.g., Siemens 840D sl with Profinet). Older controllers may not support such high-bandwidth external feedback.

Material and tool generalisation: The nominal dynamic model $\mathbf{f}_0$ was identified for AISI 1045 steel and a 10 mm ball-nose end mill. Changing the workpiece material or tool geometry would require re-identification of cutting force coefficients and possibly retuning of the neural network hyperparameters. However, the analytical CWE kernel (Steiner Roman surface) is material-independent and can be reused without modification.

Initial training: The neural network requires an initial period (≈5 s) to adapt its weights to the specific machine-tool-workpiece combination. This could be accelerated by transfer learning from previous cuts.

4 CONCLUSIONS

This paper presents a novel hybrid approach to constructing the geometric kernel of digital twins based on the use of Steiner (Roman) surfaces. The results obtained allow for the following scientifically grounded conclusions:

1.Geometric and Computational Efficiency: The proposed analytical method for determining Engagement (CWE) boundaries achieves real-time (sub-millisecond) performance without the need for iterative numerical solvers. This provides a robust foundation for implementing adaptive cutting force control at the Edge Computing level, which is critical for maintaining stability during high-speed machining.

2.Hybrid State Estimation Model: The synthesis of analytical geometry with a Lyapunov-stable neural network-augmented state estimator allows for the effective compensation of uncertain cutting dynamics (friction, thermal drift, and tool wear). Unlike purely empirical models, the proposed methodology maintains physical interpretability while ensuring high accuracy in predicting deviations.

3.Standardization and CPSMT Compatibility: The developed architecture fully complies with ISO 23218 and ISO 23704 standards, which regulate CNC system integration and the reference architecture for Cyber-Physical Smart Machine Tool systems (CPSMT). The Steiner surface-based parametrization serves as a compact information model for contact geometry, significantly simplifying implementation compared to traditional B-Rep representations.

4.Practical Value: Implementation of this approach realizes the "prediction and preservation" concept (per ISO 23704-2), enabling instantaneous detection of overloads and autonomous adaptation to tool wear. This directly enhances surface quality, extends tool life, and improves production efficiency within the Industry 4.0 and 5.0 paradigm.

Future research will focus on experimental verification across various types of machine tools, extending the methodology to additive manufacturing processes (ISO 23704-3), and standardizing the CWE information model within the ISO/TC 184/SC 1 subcommittee. Furthermore, efforts will be directed toward integrating multiple machine tools into a cloud-based digital twin for predictive maintenance and collaborative optimization.

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