THEORETICAL CALCULATION OF THE OUTFLOW VELOCITY OF AN ULTRASONICALLY GENERATED PULSATING WATER JET INCLUDING THE ACOUSTIC CONTRIBUTION OF THE SONOTRODE

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An ultrasonically generated pulsating water jet (UPWJ) can be produced by the action of an ultrasonic sonotrode on a liquid in an acoustic pressure chamber, whereby its acoustic action may affect the resulting outflow velocity. This study presents a theoretical approach to estimating the peak UPWJ outflow velocity that, in addition to hydraulic pressure, accounts for the possible acoustic contribution of the sonotrode. For the analyzed technological conditions, the estimated peak UPWJ outflow velocity was 270.74 ms^{-1}, representing an increase of approximately 7.24% compared with 252.47 ms^{-1} determined using the conventional calculation without including the acoustic contribution. The effect of this difference was subsequently assessed also through the calculated maximum impact pressure. The results indicate the potential significance of the acoustic contribution of the sonotrode in the theoretical estimation of the peak UPWJ outflow velocity and in the subsequent interpretation of its mechanical action on the material, particularly at relatively small standoff distances.

KEYWORDS

Pulsating water jet, Outflow velocity, Ultrasonic sonotrode, Acoustic pressure, Acoustic contribution, Impact pressure

1 INTRODUCTION

The outflow velocity is one of the fundamental parameters characterizing the dynamic performance of high-speed water jets. Its accurate determination is important in applications such as surface cleaning and material cutting, where the kinetic energy of the jet and the technological conditions of the process influence its interaction with the target material [Guha 2011, Krenicky 2022]. Guha et al. [Guha 2011] demonstrated the importance of jet pressure and the resulting impact conditions in water-jet cleaning, while Annoni et al. [Annoni 2008] emphasized that knowledge of the water-jet velocity distribution is fundamental for determining the cutting efficiency of water-jet and abrasive water-jet systems.

In an effort to increase the efficiency of processes utilizing water jets, pulsating water jets (PWJ) have been developed as an alternative to conventional continuous water jets (CWJ), particularly for applications involving surface cleaning and material disintegration [Sitek 2011]. Unlike CWJ, in which the liquid jet acts relatively continuously on the material surface, PWJ produces a temporally discontinuous, or periodically modulated, action of the liquid on the target surface. The increased efficiency of PWJ is associated with the breakup of the continuous jet into individual liquid clusters, whose repeated impact on the surface leads to the periodic generation of short-duration pressure peaks [Foldyna 2009]. During high-velocity liquid impact, rapid deceleration of the liquid can result in very high local pressure loads, which are significant in terms of the erosive effect of the liquid jet [Cook 1928, Heymann 1970]. In the case of a pulsating jet, such impact events are periodically repeated, thereby changing the nature of the jet–surface interaction compared with CWJ. The high short-duration impact pressures generated during individual impacts thus represent one of the physical mechanisms that contribute to the increased erosive effect of PWJ [Foldyna 2018].

Pulsating water jets can be generated in various ways, with the principles used including, for example, mechanical interruption of CWJ [Yamagata 2023], the use of self-resonating nozzles/heads [Feng 2024, Ma 2024, Riha 2026], or the application of ultrasonic excitation. In the case of ultrasonically excited pulsating water jets (UPWJ), jet pulsation can be generated, for example, using an ultrasonic nozzle [Xiong 2024] or by means of an ultrasonic sonotrode [Foldyna 2006a], whose periodic oscillatory motion induces pressure pulsations in the acoustic chamber, leading to variations in the outflow velocity in the axial direction and the subsequent gradual breakup of the jet into individual pulses [Foldyna 2011], see Fig. 1.

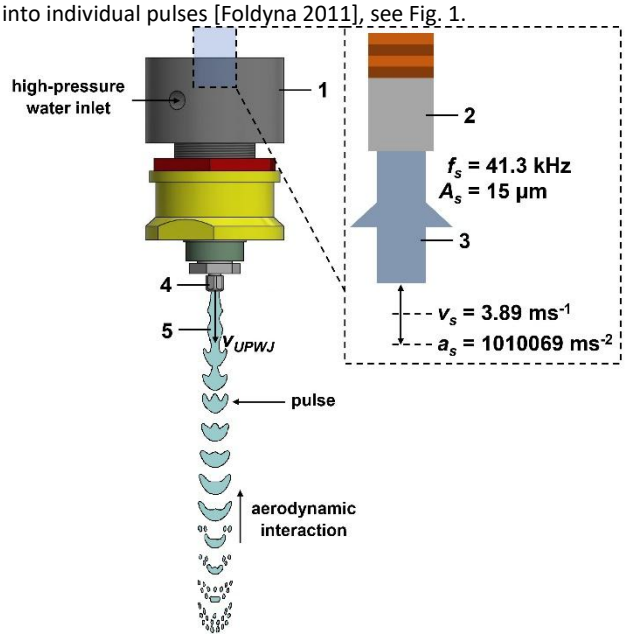


Figure 1. Illustration of the UPWJ head design with an ultrasonic sonotrode and the structure of the ultrasonically generated pulsating water jet (1 – UPWJ head, 2 – ultrasound head with piezoelectric transducer, 3 – sonotrode, 4 – nozzle, 5 – high-frequency UPWJ)

Regardless of the method used to generate PWJ, however, its effect on the target material is also related to the velocity at which the individual liquid clusters impact its surface. As with CWJ, the impact velocity therefore represents an important parameter characterizing the conditions of interaction between PWJ and the material [Foldyna 2004]. Knowledge of the outflow or impact velocity is therefore important when evaluating the erosion efficiency of PWJ. In the case of UPWJ generated by an ultrasonic sonotrode, however, the outflow velocity is not determined solely by the water pressure generated by the high-pressure pump. The periodic motion of the sonotrode represents an additional source of mechanical energy that is transferred to the liquid through the acoustic field. Nevertheless, in previous experimental studies focused on the erosion efficiency of UPWJ generated by an ultrasonic sonotrode, only a simplified approach has been applied to

calculate the outflow velocity, based solely on the hydrodynamic pressure generated by the high-pressure pump – this model neglected the mechanical contribution of the oscillating sonotrode to the overall energy balance of the system [Coranic 2023]. The consequences of such a simplification may not be limited to the determination of the outflow velocity itself but may also affect the subsequent interpretation of the UPWJ erosion mechanism. An example is the study by Nag et al. [Nag 2021], in which the water-hammer effect was considered in the analysis of the erosive effect of UPWJ, and the impact pressure was derived from the liquid outflow velocity for CWJ. Since the impact pressure is directly dependent on the outflow/impact velocity [Heymann 1969], inaccurate determination of this velocity may also lead to inaccurate determination of the intensity of individual impacts. When evaluating the effect of UPWJ frequency on tantalum erosion, the authors observed an increase in erosion depth when the frequency was increased from 20 to 40 kHz at the same pressure, nozzle diameter, and exposure time. This difference may be related not only to a higher number of impacts per unit time, but potentially also to a change in the intensity of individual impacts if a change in the ultrasonic excitation conditions affects the acoustic contribution to the outflow and impact velocities. A more accurate determination of the outflow velocity and the impact pressure derived from it could therefore enable a more precise distinction between the effect of frequency in terms of the number of impacts and its effect in terms of their intensity. A similar question also arises when interpreting the results of the study by Hloch et al. [Hloch 2020], in which the effect of UPWJ operating pressure on the erosive effect at different standoff distances (5–101 mm) was investigated. In this study, increasing the pressure (within the range of 40–100 MPa) was associated with a change in the intensity of the jet action and the resulting erosive effect (groove depth). In UPWJ generated by an ultrasonic sonotrode, however, a change in hydraulic pressure may not represent only a change in the energy supplied by the high-pressure pump. If the proportion of the acoustic contribution of the sonotrode to the resulting outflow velocity also changes with pressure, then the change in impact intensity may not be unequivocally attributable solely to the change in hydraulic pressure. A more accurate determination of the outflow velocity and the impact pressure derived from it could therefore enable the effect of hydraulic pressure to be better distinguished from the possible contribution of the acoustic energy of the sonotrode, thereby allowing a more accurate interpretation of the effect of pressure on the erosive effect of UPWJ. The above considerations may also be relevant to the interpretation of the results reported by Poloprudsky et al. [Poloprudsky 2024], who investigated the effects of standoff distance and impact distribution on the erosion efficiency of UPWJ at hydraulic pressures of 30, 40, 50, and 60 MPa. The authors found that decreasing the impact distribution resulted in a decrease in the investigated parameters of the erosive effect (Ra – arithmetic mean height, Rz – maximum height, Rv – mean pit depth), with this decrease being less pronounced at higher hydraulic pressure levels. In interpreting the impact effect, the authors used the jet outflow velocity corresponding to CWJ. However, if the UPWJ outflow velocity were determined by taking into account the acoustic contribution of the sonotrode, the resulting change in the determined velocity would also be directly reflected in the impact pressure. A higher actual outflow or impact velocity, with the liquid density and acoustic velocity remaining unchanged, would lead to a higher calculated impact pressure. The observed smaller decrease in erosion efficiency with decreasing impact distribution as the hydraulic pressure increased might therefore not be related solely to the number of impacts. If the actual UPWJ impact velocity also changes with increasing hydraulic pressure, the intensity of individual impacts, which can be expressed through the impact pressure, may change simultaneously. Without an accurate determination of the outflow velocity and the impact pressure derived from it, it is therefore not possible to determine unequivocally to what extent the observed erosive effect results from the number of impacts and to what extent from their individual intensity.

The above considerations indicate the need for a more accurate determination of the outflow velocity of UPWJ generated by an ultrasonic sonotrode, taking into account not only the hydraulic pressure but also the acoustic contribution of the sonotrode to the energy balance of the system, since this contribution may not only affect the axial modulation of the UPWJ velocity but may also contribute to an increase in its resulting outflow velocity. This raises the question of the extent to which the outflow velocity of UPWJ generated by an ultrasonic sonotrode may differ from the velocity determined solely on the basis of hydraulic pressure and what proportion of this velocity is attributable to the energy transferred to the liquid by the acoustic sonotrode. The aim of this study is therefore to present a theoretical model for estimating the peak UPWJ outflow velocity while taking into account the acoustic contribution of the sonotrode and, on this basis, to assess the difference between the outflow velocity determined using the conventional approach based solely on hydraulic pressure and the velocity that also accounts for the acoustic contribution. The velocity determined in this way may subsequently provide a more accurate basis for determining the impact velocity and impact pressure, and thus for a more accurate interpretation of the physical conditions of UPWJ action on the target material.

2 THEORETICAL DETERMINATION OF UPWJ OUTFLOW VELOCITY

The following section presents a model theoretical procedure for estimating the peak UPWJ outflow velocity while taking into account the acoustic contribution of the ultrasonic sonotrode. The calculation is performed for selected values of input parameters characterizing the oscillatory motion of the sonotrode, particularly its frequency and amplitude, as well as other parameters of the liquid and high-pressure system required for the individual calculation steps. The values of the input parameters are selected to correspond to conditions actually used in the experimental generation of UPWJ, so that the model calculation is based on experimentally relevant conditions. In addition to the quantities directly involved in determining the resulting outflow velocity, selected associated parameters are also calculated within the procedure. These parameters are not essential for the calculation of the outflow velocity itself, but complement the physical description of the oscillatory motion of the sonotrode and the related processes. Their inclusion enables the individual calculation steps to be presented clearly while also providing a more comprehensive characterization of the physical parameters of the system under consideration.

2.1 Kinematic parameters of the oscillatory motion of the sonotrode

The sonotrode performs a harmonic oscillatory motion in the pressure chamber, which introduces periodic pressure variations into the liquid. For selected parameters (sonotrode frequency f_s = 41300 Hz, sonotrode amplitude A_s = 0.000015 m), it is possible to calculate its maximum attainable velocity and acceleration.

For the velocity v_s of the sonotrode, the following relationship applies:

$$\boldsymbol{v_s = \omega A_s \cos(\omega t)} \quad (1)$$

where:
ω – angular frequency of the oscillatory motion of the sonotrode [s^{-1}],
t – time (at the equilibrium position, the time t = 0 s is considered).
For the angular frequency ω, the following relationship applies:

$$\boldsymbol{\omega = 2\pi f_s} \quad (2)$$

$$\boldsymbol{\omega = 2 \cdot \pi \cdot 41300\ Hz = 259495.55\ s^{-1}} \quad (3)$$

By substituting into Equation (1), the velocity v_s of the sonotrode at the equilibrium position is obtained:

$$\boldsymbol{v_s = 259495.55\ s^{-1} \cdot 0.000015\ m \cdot \cos(259495.55\ s^{-1} \cdot 0\ s) = 3.89\ ms^{-1}} \quad (4)$$

For the calculation of the maximum acceleration a_s of the sonotrode at the extreme position, the following relationship applies:

$$\boldsymbol{a_s = \omega^2 A_s \sin(\omega t)} \quad (5)$$

First, it is necessary to calculate the unknown value of time t, i.e., the time at which the sonotrode reaches its extreme position. The basis is the equation for the instantaneous displacement of harmonic oscillatory motion:

$$\boldsymbol{y_s = A_s \sin(\omega t)} \quad (6)$$

where:
y_s – instantaneous displacement of the sonotrode at time t (i.e., in this case $y_s = A_s$ = 0.000015 m).
By subsequently substituting the values into Equation (6) for the instantaneous displacement, the time t at which the sonotrode is located at the extreme position is derived:

$$\boldsymbol{0.000015\ m = 0.000015\ m \cdot \sin(259495.55\ s^{-1} \cdot t)} \quad (7)$$

$$\boldsymbol{\sin(259495.55\ s^{-1} \cdot t) = 1} \quad (8)$$

$$\boldsymbol{\sin(259495.55\ s^{-1} \cdot t) = \sin\frac{\pi}{2}} \quad (9)$$

$$\boldsymbol{259495.55\ s^{-1} \cdot t = \frac{\pi}{2}} \quad (10)$$

$$\boldsymbol{t = \frac{\pi}{2 \cdot 259495.55\ s^{-1}} = 0.000006053\ s} \quad (11)$$

By substituting into Equation (5) for the acceleration a_s of the sonotrode at the extreme position, the following relationship applies:

$$\boldsymbol{a_s = (259495.55\ s^{-1})^2 \cdot 0.000015\ m \cdot \sin(259495.55\ s^{-1} \cdot 0.000006053\ s)} \quad (12)$$

$$\boldsymbol{a_s = 1010069.105\ ms^{-2}} \quad (13)$$

Considering the oscillating sonotrode within the system and the quantities describing its motion, the following theoretical calculation has been derived.

2.2 Determination of the actual liquid density ρ

The initial step is the determination of the water density ρ at the selected operating pressure (p = 40 MPa). Unlike the reference density ρ_{ref} at atmospheric pressure, volumetric compression of the liquid occurs in the system. According to Foldyna et al. [Foldyna 2007], the density is a function of the pressure p and the bulk modulus of elasticity K:

$$\boldsymbol{\rho = \frac{\rho_{ref}}{1 - \frac{p - p_{ref}}{K}}} \quad (14)$$

where:
ρ_{ref} = 998.2 kg·m^{-3} (reference density of water),
p = 40·10^6 Pa (operating static pressure),
p_{ref} = 101325 Pa (reference (atmospheric) pressure),
K = 2.2·10^9 Pa (bulk modulus of elasticity of water).
By substituting the values into Equation (14):

$$\boldsymbol{\rho = \frac{998.2\ kgm^{-3}}{1 - \frac{40 \cdot 10^6\ Pa - 101325\ Pa}{2.2 \cdot 10^9\ Pa}}} \quad (15)$$

$$\boldsymbol{\rho = 1016.64\ kgm^{-3}} \quad (16)$$

2.3 Calculation of the acoustic wave propagation velocity a

The speed of sound in water a is not constant but increases with the applied pressure. For the purposes of an accurate calculation of acoustic energy transmission in the pressure system, it is necessary to consider this dependence. In accordance with experimental measurements and the so-called "UNESCO equation", an approximation relationship can be used for the pressure p [Foldyna 2007, Foldyna 2006b]:

$$\boldsymbol{a = 2 \cdot 10^{-6} p + 1432} \quad (17)$$

$$\boldsymbol{a = 2 \cdot 10^{-6} \cdot 40 \cdot 10^6\ Pa + 1432 = 1512\ ms^{-1}} \quad (18)$$

2.4 Quantification of the acoustic contribution of the sonotrode and modulation of the resulting pressure in the nozzle

The oscillating sonotrode acts on the water as a source of acoustic waves, i.e., the mechanical energy of the sonotrode oscillations is transferred into the form of pressure pulsations (acoustic waves) in a compressible medium. The amplitude of the acoustic pressure Δp_{ac} induced by the motion of the sonotrode in a compressible liquid is determined by the product of the specific acoustic impedance of the liquid ($\rho \cdot a$) and the maximum velocity of the oscillating body surface (i.e., the sonotrode surface), v_s [Kinsler 2000]:

$$\boldsymbol{\Delta p_{ac} = \rho a v_s} \quad (19)$$

$$\boldsymbol{\Delta p_{ac} = 1016.64\ kgm^{-3} \cdot 1512\ ms^{-1} \cdot 3.89\ ms^{-1} = 5.98\ MPa} \quad (20)$$

The calculated value of Δp_{ac} (Equation 20) represents the amplitude of the acoustic pressure generated by the sonotrode directly at its front face. In the real UPWJ system, this wave propagates through the liquid waveguide, where the superposition of direct and reflected waves results in the formation of a standing acoustic wave. The internal geometry of the acoustic chamber acts as a mechanical amplifier, while an appropriate tuning of the chamber length L_c leads to a progressive increase in the pressure amplitude towards the outlet opening (nozzle). The maximum acoustic pressure is therefore achieved in the critical region of the nozzle, where the pressure energy is transformed into velocity pulsations [Foldyna 2007]. Since the actual acoustic pressure amplitude in the nozzle region is not known, an approximate value of Δp_{ac} = 6 MPa, based on the amplitude calculated at the sonotrode surface, is considered for the subsequent theoretical calculation of the peak UPWJ outflow velocity. This approach represents a simplifying assumption that does not account for possible changes in the acoustic pressure amplitude during its propagation through the acoustic chamber. The actual amplitude of pressure oscillations in the nozzle region may therefore differ from the assumed value, which represents one of the sources of uncertainty in the subsequent theoretical estimation of the peak UPWJ outflow velocity.
Following the determined amplitude of the acoustic pressure Δp_{ac} (Equation 20), it is possible to define the total instantaneous pressure acting in the critical cross-section of the nozzle. This

pressure is not a constant quantity in the UPWJ system but, due to the introduced acoustic energy, periodically oscillates around the mean value of the static pressure p generated by the high-pressure pump. The resulting pressure at the nozzle location is at each moment determined by the superposition of both components, while its maximum value at the pulse peak p_{max} reaches the following level at the moment of the positive amplitude of the acoustic wave:

$$\boldsymbol{p_{max} = p + \Delta p_{ac}} \tag{21}$$

$$\boldsymbol{p_{max} = 40\ MPa + 6\ MPa = 46\ MPa} \tag{22}$$

2.5 Calculation of the estimated peak UPWJ outflow velocity

The maximum instantaneous pressure p_{max} subsequently enters the calculation of the estimated peak UPWJ outflow velocity (v_{UPWJ}) according to the Bernoulli equation, while considering the compressibility and actual density of the liquid. This calculation is based on a quasi-steady approximation, which assumes that the instantaneous outflow velocity corresponds to the instantaneous pressure in the nozzle region according to a relationship valid for steady flow. However, at the acoustic excitation frequency (f_s = 41.3 kHz), the dynamic response of the flow may be influenced by the inertial effects of the liquid, the propagation and reflection of acoustic waves, as well as a possible phase shift between pressure and velocity oscillations. Therefore, the applied calculation procedure provides a simplified theoretical estimate of the peak UPWJ outflow velocity, the accuracy of which depends on the dynamic characteristics of the specific high-pressure system and requires experimental verification:

$$\boldsymbol{v_{UPWJ} = \sqrt{\frac{2p_{max}}{\rho}}} \tag{23}$$

$$\boldsymbol{v_{UPWJ} = \sqrt{\frac{2 \cdot 46 \cdot 10^6\ Pa}{1016.64\ kgm^{-3}}} = 300.82\ ms^{-1}} \tag{24}$$

To account for deviations of the actual outflow velocity from the ideal value determined using the Bernoulli equation, an assumed velocity coefficient C_v = 0.9 is introduced in the presented theoretical model and applied directly to the calculated ideal outflow velocity (Equation 25). This value has not been experimentally determined for the investigated high-pressure system and represents a simplifying assumption of the model. However, it should be noted that ultrasonic excitation may influence the flow characteristics within the nozzle, including the possible occurrence of cavitation and time-dependent changes in flow characteristics depending on the phase of the pressure oscillations. The possible time dependence of the velocity coefficient is therefore not accounted for in the presented model.

$$\boldsymbol{v_{UPWJ} = 300.82\ ms^{-1} \cdot 0.9 = 270.74\ ms^{-1}} \tag{25}$$

The value thus obtained (Equation 25) represents a theoretical estimate of the peak UPWJ outflow velocity corresponding to the positive peak of the acoustic pressure oscillation, while no increase in the time-averaged outflow velocity can be inferred from this calculation.

The theoretical outflow velocity of the jet (v_{CWJ}) at the same static pressure p = 40 MPa, when applying the simplified model without the influence of ultrasound (CWJ) and liquid compressibility, would reach:

$$\boldsymbol{v_{CWJ} = \sqrt{\frac{2 \cdot 40 \cdot 10^6\ Pa}{998.2\ kgm^{-3}}} = 283.1\ ms^{-1}} \tag{26}$$

When calculating the theoretical CWJ outflow velocity, the same assumed velocity coefficient C_v = 0.9 as for UPWJ is considered to account for the non-ideal nature of the flow (Equation 27). This approach allows the calculated velocities to be compared under the same assumption regarding deviations from ideal flow. However, the actual values of the velocity coefficient may differ between the UPWJ and CWJ operating modes, and such potential differences are not accounted for in the presented theoretical model.

$$\boldsymbol{v_{CWJ} = 283.1\ ms^{-1} \cdot 0.9 = 254.79\ ms^{-1}} \tag{27}$$

The comparison of the calculated values shows that, under the given technological conditions, the estimated peak UPWJ outflow velocity (v_{UPWJ} = 270.74 ms^{-1}) is approximately 6.26% higher than the theoretical outflow velocity v_{CWJ} = 254.79 ms^{-1} determined using the simplified calculation model. However, this difference is not attributable exclusively to the possible acoustic contribution of the sonotrode, but also includes the effect of the different liquid densities considered in the respective calculations. This comparison thus highlights the difference that may arise when determining the UPWJ outflow velocity if both the possible acoustic contribution of the sonotrode and the change in liquid density due to the operating pressure are neglected. Such a simplified approach to determining the outflow velocity of UPWJ generated by an ultrasonic sonotrode, based exclusively on hydraulic pressure while assuming the reference liquid density, has commonly been used in experimental studies focused on evaluating its erosion effects (examples of some of these studies have already been presented above, while others are discussed in the following section of the article).

To separately assess the effect of the possible acoustic contribution of the sonotrode on the resulting outflow velocity, a comparison must be performed using the same liquid density. If the same density as for UPWJ, i.e., ρ = 1016.64 kg·m^{-3}, corresponding to the operating pressure p = 40 MPa, is considered when calculating the outflow velocity v_{CWJ}, we obtain:

$$\boldsymbol{v_{CWJ} = \sqrt{\frac{2 \cdot 40 \cdot 10^6\ Pa}{1016.64\ kgm^{-3}}} = 280.52\ ms^{-1}} \tag{28}$$

After applying the velocity coefficient C_v = 0.9:

$$\boldsymbol{v_{CWJ} = 280.52\ ms^{-1} \cdot 0.9 = 252.47\ ms^{-1}} \tag{29}$$

By comparing the outflow velocity v_{CWJ} determined in this way (Equation 29) with the UPWJ outflow velocity (v_{UPWJ} = 270.74 ms^{-1}), a difference of approximately 7.24% is obtained. Since the same liquid density and velocity coefficient were considered in both calculations, the observed difference can be attributed exclusively to the possible acoustic contribution of the sonotrode within the presented theoretical model, whose additional energy manifests itself in an increase in the maximum instantaneous pressure in the nozzle region and consequently in the UPWJ outflow velocity. The trend obtained from the theoretical calculations can be considered physically consistent with the experimental observations reported by Zelenak et al. [Zelenak 2015]. Using optical diagnostics combined with PIV analysis, the authors demonstrated that, at all investigated operating pressures (p = 8, 12, and 20 MPa), the instantaneous velocities of the UPWJ measured in the region immediately downstream of the nozzle outlet reached higher values than those observed for the corresponding CWJ. Although this difference between the UPWJ and CWJ velocities was experimentally observed, its physical origin was not further analysed in the study. In particular, the authors did not investigate whether the higher instantaneous velocities of the UPWJ could be associated with the additional acoustic energy introduced into the pressurised liquid by the oscillating sonotrode. Consequently, although the occurrence of higher instantaneous UPWJ velocities compared with CWJ had already

been experimentally documented, the mechanism responsible for this increase and its potential relationship with the acoustic contribution of the sonotrode remained outside the scope of the investigation.

2.6 Calculation of impact pressure

Based on the above considerations, the effect of the calculated difference between the UPWJ and CWJ outflow velocities can also be demonstrated in terms of the impact pressure (p_i) generated under the given conditions. According to Heymann [Heymann 1969], the maximum instantaneous impact pressure generated during the high-speed impact of a liquid drop on a rigid solid surface can be approximated for water by:

$$\boldsymbol{p_i = 3\rho_0 C_0 v} \tag{30}$$

where v is the impact velocity of the liquid, ρ_0 is the density of the liquid at ambient pressure, and C_0 is the acoustic velocity in the undisturbed liquid. This approximation is applicable to water within the impact Mach number range $0{,}03 \leq v/C_0 \leq 0{,}3$.

When the calculated outflow velocities are considered as characteristic impact velocities of individual liquid elements, and the same values of the liquid properties are assumed for both cases (ρ_0 = 998.2 kg·m^{-3} and C_0 = 1482 ms^{-1}), the theoretical maximum impact pressures corresponding to the CWJ and UPWJ velocities can be expressed as:

$$\boldsymbol{p_{i(CWJ)} = 3\rho_0 C_0 v_{CWJ}} \tag{31}$$

$$\boldsymbol{p_{i(CWJ)} = 3 \cdot 998.2\ kgm^{-3} \cdot 1482\ ms^{-1} \cdot 252.47\ ms^{-1} = 1120.46\ MPa} \tag{32}$$

$$\boldsymbol{p_{i(UPWJ)} = 3\rho_0 C_0 v_{UPWJ}} \tag{33}$$

$$\boldsymbol{p_{i(UPWJ)} = 3 \cdot 998.2\ kgm^{-3} \cdot 1482\ ms^{-1} \cdot 270.74\ ms^{-1} = 1201.54\ MPa} \tag{34}$$

Thus, the increase in the theoretical outflow velocity from 252.47 ms^{-1} for CWJ to 270.74 ms^{-1} for UPWJ results in an increase in the calculated maximum impact pressure from 1120.46 MPa to 1201.54 MPa, corresponding to an increase of approximately 7.24%. Since the Heymann approximation is linearly dependent on the impact velocity, the relative increase in the calculated maximum impact pressure directly reflects the calculated relative increase in the outflow velocity. Although the increase in the calculated maximum impact pressure is approximately 7.24%, in absolute terms it corresponds to a difference of approximately 81 MPa, which may not be negligible in terms of the mechanical action of the jet on the impacted material. Such a difference may become significant when assessing the mechanical response of the impacted material, particularly in relation to its yield strength and ultimate tensile strength. If the stresses induced in the material by liquid impact approach or exceed the corresponding strength characteristics of the material, even a relatively moderate increase in impact pressure may contribute to more pronounced plastic deformation and, under repeated loading, to the initiation and subsequent development of material damage. From this perspective, the additional acoustic contribution included in the calculation of the UPWJ outflow velocity may have implications extending beyond the increase in jet velocity itself. The resulting increase in the calculated maximum impact pressure suggests that neglecting the acoustic contribution may also lead to an underestimation of the mechanical loading to which the impacted material may be subjected.

The above difference in the calculated maximum impact pressure resulting from different outflow velocity values may be particularly relevant in experiments conducted at relatively small standoff distances, where it can be assumed that the velocity of the liquid elements upon impact remains relatively close to their velocity in the nozzle outlet region. These considerations may be relevant, for example, when interpreting the results of the study by Nag et al. [Nag 2023], in which UPWJ experiments were conducted at standoff distances of 20–52 mm, and the significantly higher erosive effect of UPWJ compared with CWJ was interpreted primarily as a consequence of repeated impacts of water clusters, which generate pressure stresses followed by radial flow inducing shear stresses. However, the possible increase in the UPWJ outflow velocity due to acoustic excitation by the sonotrode was not taken into account in the calculation of the impact pressure, although the authors themselves state that the pressure fluctuations generated by the sonotrode are transformed into velocity fluctuations at the nozzle outlet, leading to the formation of faster- and slower-moving parts of the jet. When this acoustic contribution is taken into account, the higher erosive effect of UPWJ could therefore be interpreted not only as a consequence of the repeated action of individual impacts, but potentially also as a consequence of the higher instantaneous impact velocity of the liquid elements and, consequently, the higher impact pressure of each such impact. A similar aspect can also be identified in the study by Poloprudsky et al. [Poloprudsky 2022], in which theoretical UPWJ velocities of 184 and 260 ms^{-1} were used at pressures of 20 and 40 MPa, respectively, determined on the basis of hydraulic conditions and a discharge coefficient C_d = 0.92, while the experiments were conducted at relatively small standoff distances of 16 and 32 mm. The acoustic contribution of the sonotrode to the velocity of the individual liquid elements was not taken into account when determining these velocities. This fact may also be significant when interpreting residual stresses. The authors found that compressive residual stresses exhibited a relatively low dependence on the number of impacts but a high dependence on water pressure, while reaching higher values for UPWJ than for CWJ. Since increasing the pressure simultaneously increases the hydraulically determined jet velocity, this trend can be interpreted primarily as a consequence of the higher intensity of individual impacts rather than solely their repeated action. However, if the additional acoustic energy of the sonotrode were also taken into account when determining the impact velocity, changes could occur not only between the impact velocities of UPWJ and CWJ at the same hydraulic pressure, but the difference between the UPWJ impact velocities at 20 and 40 MPa could also change. Consequently, the differences in the intensity of individual impacts (in impact pressures) between the two UPWJ pressure levels could also change. The higher compressive residual stresses observed for UPWJ at 40 MPa might therefore not be related exclusively to the increase in hydraulic pressure and the associated hydraulically determined velocity, but potentially also to a change in the acoustic contribution to the resulting impact velocity of the liquid elements.

Based on the above considerations, it therefore appears appropriate to take into account the possible acoustic contribution of the oscillating sonotrode when theoretically determining the outflow velocity of UPWJ generated by an ultrasonic sonotrode. The outflow velocity determined in this way may provide a more comprehensive basis for the subsequent estimation of the impact velocity and impact pressure, and thus also for interpreting the mechanical action of UPWJ on the impacted material. Taking this contribution into account is appropriate not only in experiments with normal UPWJ impact but also in experiments with oblique impact, in which the impact angle significantly affects the nature of the interaction between UPWJ and the material [Cuha 2022].

3 CONCLUSIONS

In the present study, a theoretical approach to estimating the peak outflow velocity of UPWJ generated by an acoustic sonotrode was presented, which, unlike the conventional calculation based exclusively on hydraulic pressure, also takes into account the possible acoustic contribution of the oscillating sonotrode. This approach is based on the assumption that the mechanical oscillations of the sonotrode transmitted into the pressurized liquid represent an additional energy input that manifests itself through modulation of the axial velocity of the emerging jet in such a way that the maximum outflow velocity may be higher than that of a CWJ system at the same hydraulic pressure.

For the analyzed technological conditions (p = 40 MPa, f_s = 41.3 kHz, A_s = 15 µm, C_v = 0.9), the estimated peak UPWJ outflow velocity determined by including the acoustic contribution was v_{UPWJ} = 270.74 ms^{-1}, representing an increase of approximately 7.24% compared with v_{CWJ} = 252.47 ms^{-1} determined solely from the hydraulic conditions. When the Heymann approximation was applied, this difference resulted in an increase in the maximum impact pressure from 1120.46 MPa to 1201.54 MPa, i.e., by approximately 81 MPa. The results highlight the importance of taking into account the possible acoustic contribution of the sonotrode when theoretically determining the UPWJ outflow velocity and subsequently interpreting its action on the material, particularly at relatively small standoff distances.

However, the presented calculation approach represents a simplified theoretical model, the results of which are subject to several assumptions, particularly the assumed acoustic pressure amplitude in the nozzle region, the application of the quasi-steady approximation, and the assumption of the same constant velocity coefficient for UPWJ and CWJ. Future research should therefore focus primarily on the direct measurement of the instantaneous UPWJ velocity near the nozzle outlet while simultaneously monitoring the acoustic excitation parameters, and on comparing the measured values with the velocities predicted by the presented model. Such verification would enable a more accurate quantification of the actual contribution of the acoustic energy of the sonotrode to the resulting UPWJ dynamics, an assessment of the validity of the simplifying assumptions applied, and a determination of the range of validity of the presented approach under different technological conditions.

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