

ACCURACY EVALUATION AND VALIDATION OF A DEEP LEARNING SURROGATE MODEL FOR RAYLEIGH STEP BEARING

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Hydrodynamic fluid bearings have become increasingly sophisticated, and the resulting design requirements have gradually exceeded the capabilities of conventional exhaustive analytical approaches. This development has increased interest in alternative methods, including data-driven modeling. Although black-box surrogate modeling requires evaluation of the physical validity of its predicted solutions, assessing problems that lack theoretical solutions is extremely difficult. In this study, a surrogate deep learning model was developed for Rayleigh step bearing, for which the analytical solution is known, and its predictive accuracy was quantitatively evaluated. The results presented that, when the dataset was sufficiently large, the correlation coefficient between the prediction error rate on the validation data, which constituted part of the dataset, and the prediction error rate across the entire theoretical solution distribution was high. The model also achieved a practically acceptable approximation accuracy when Gaussian noise was intentionally added to the training data. These findings demonstrate the effectiveness and robustness of the proposed surrogate models and further present that the Rayleigh step bearing can function as a benchmark tool for verifying the methodological validity of various evaluation approaches.

KEYWORDS

Hydrodynamic fluid bearing, Optimization design, Rayleigh step bearing, Deep learning, Surrogate modelling

1 INTRODUCTION

A thrust bearing is an essential machine element that supports axial loads in turbomachinery and similar equipment. Types of hydrodynamic fluid bearings include grooved [Malanowski 1965], [Hashimoto 2008], [Miyana 2016] and tilting-pad types [Lund 1964], [Hargreaves 2007]. Among these types, the step-type bearing is recognized as a comparatively high load-capacity configuration. Bearings incorporating texturing techniques such as dimples have also begun to attract attention in recent years [Gropper 2018], [Marian 2007], [Miyana 2025], [Ishii 2026], [Ishii 2023], [Henry 2018]. Most of these techniques aim to improve lubrication performance by modifying the bearing surface topology, making the optimization of bearing geometry an important issue in tribology. Among these systems, Rayleigh step bearing is one of the few cases for which optimal solutions for both the film thickness ratio and the step position can be

determined. Furthermore, the analytical solution to this system is well known and remains widely used for practical initial design evaluations [Xiaodong 2024], [Kumar 2019].

Meanwhile, demands on bearing design are becoming increasingly advanced and multi-objective, and the higher dimensionality and nonlinear nature of systems often make conventional analytical techniques and exhaustive searches difficult to apply. Given this background, data-driven approaches, such as surrogate modeling, are attracting attention as methods to reduce evaluation costs and efficiently explore design spaces. Conventional optimization based on physical models required converting physical phenomena into mathematical models. In contrast, data-driven methods identify the relationship between inputs and outputs using a surrogate model, eliminating the need for complex physical modeling and enabling rapid optimization. Even in recent tribology research, examples of applications have appeared, particularly in wear prediction [Argatov 2019], [König 2024], [Zhu 2024], [Wattel 2026] and lubrication condition assessment [Sinanoğlu 2004], [Baş 2023], [Marian 2023], [Granja 2026].

Nevertheless, fundamental challenges remain in data-driven approaches. Validation of the solutions predicted by surrogate models is always essential, but many studies address complex problems that lack analytical solutions. This issue makes it consistently difficult to verify whether the obtained solutions are valid quantitatively. In lubrication problems, this concern is expected to become even more serious in cases involving phenomena such as cavitation. In other words, the governing equations at gas-liquid interfaces exhibit significant nonlinearity and discontinuity. This poses a substantial risk that surrogate models may learn nonphysical local solutions.

From this perspective, the Rayleigh step bearing, whose analytical solution is known, is of great significance. This problem is not merely a theoretical exercise, and can also be interpreted as a physical benchmark for verifying whether deep learning models are conducting physically valid exploration. Furthermore, this system is low-dimensional, so the search process and error propagation can be easily visualized, clearly revealing the behavior of optimization algorithms. Direct comparison with true values also enables the quantitative evaluation of the data density required to avoid nonphysical local solutions and the limits of physical reproducibility for each algorithm.

Given the above-mentioned background, this study applies surrogate modeling to the load-capacity maximization problem for the Rayleigh step bearing, using the step position and film thickness ratio as design variables, and directly compares the model outputs with the known analytical solutions. The results of this study provide a fundamental baseline for reliably applying deep learning to complex lubrication systems that lack analytical solutions.

2 RESEARCH METHOD

2.1 Rayleigh Step bearing

The step bearing is a stepped parallel bearing, and Rayleigh identified the step position and step ratio that yield the maximum load capacity [Rayleigh 1918]. This bearing is used in applications such as high-speed turbomachinery and MEMS. Figure 1 presents a schematic illustration of the Rayleigh step bearing, which is a one-dimensional step bearing. When the assumption of infinite width is adopted, the Reynolds equation is expressed as Equation (1).

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\eta} \frac{\partial p}{\partial x} \right) = 6U \frac{\partial h}{\partial x} \quad (1)$$

$$h = \begin{cases} h_i & | 0 \leq x \leq B_i \\ h_o & | B_i < x \leq B \end{cases}$$

where x is the coordinate, h is the film thickness, η is the viscosity, and U is velocity of the moving plane.

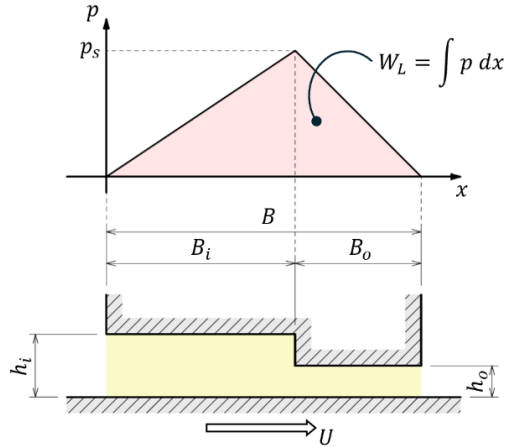


Figure 1. Schematic illustration of Rayleigh step bearing

Given that the film thickness is constant in both the inlet and outlet regions, the pressure distribution increases linearly from the inlet to the step section, then decreases linearly from the step section to the outlet.

Considering continuity of flow rate at the step position, the following Equation (2) gives the pressure at step position applies:

$$P_s = \frac{\eta U B}{h_o^2} \frac{3k(1-k)(m-1)}{k + (1-k)m^3} \quad (2)$$

The following Equation (3) gives the load capacity per unit width:

$$W_L = \frac{\eta U B^2}{h_o^2} \frac{3k(1-k)(m-1)}{k + (1-k)m^3} \quad (3)$$

where $k(= B_i/B_o)$ denotes the step length ratio, and $m(= h_i/h_o)$ denotes the film thickness ratio.

In this equation, the first part on the right-hand side represents the operating conditions, and the latter part represents the geometric parameters of the surface. This study maintains constant operating conditions and focuses on the geometric parameters of the step bearing. If the geometric parameter term is defined as $X(m, k)$ in Equation (4), then the optimization problem for W_L can be reformulated as an optimization problem for X .

$$X(m, k) = \frac{3k(1-k)(m-1)}{k + (1-k)m^3} \quad (4)$$

Figure 2 shows the solution distribution of X . At $m = 1$ and $k = 0, 1$, no step exists, and no wedge-film effect occurs. These results in $X = 0$. Rayleigh also solved this expression numerically to demonstrate that the optimum value is $X_{max}|_{(m,k)=(1.87,0.718)} = 0.20626$ [Rayleigh 1918].

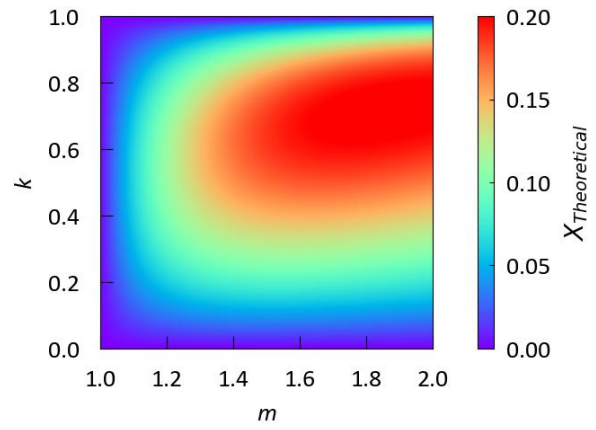


Figure 2. X of Equation (4)

2.2 Deep learning model

The replacement of $X(m, k)$ is considered using a deep learning model. Therefore, it is sufficient to construct a model in which m and k are supplied as inputs, computations are performed through several hidden layers, and X is produced as the output.

A major issue in implementing a regression model such as the present one lies in determining the hyperparameters. Hyperparameters include numerous settings such as the number of hidden layers and the number of nodes, and their selection is generally based on empirical knowledge. Automatic optimization frameworks such as Optuna [Akiba 2019] have recently simplified this process, but the search range must still be defined for each model, meaningful automation has not yet been achieved. Therefore, a thorough understanding of the influence of various parameters is essential for improving approximation accuracy.

The present study adopted a fully connected neural network, the most basic form of deep learning model, in which every node in each layer is connected to all nodes in the adjacent layers. Additionally, implementation was carried out using the Keras framework. Table 1 lists the various hyperparameters. Testing could be performed against the theoretical solution distribution, so only training and validation data need to be prepared as datasets. Given this, three datasets of different sizes, namely 121, 441, and 1 681, were created by discretizing m and k with different intervals. During training, 20% of each dataset was used as validation data, and the remainder was used as training data. For the test data, a sufficiently large dataset was prepared relative to the dataset size to represent the solution distribution, yielding 1 002 001 data points at intervals of 0.001. Gaussian noise G was also added to the datasets, and training effectiveness on error-containing data was examined. This assumes training with data that exhibit variability similar to that found in experimental measurements. The noise values followed a normal distribution with a standard deviation $\sigma = G_{max}/3$ relative to the dataset mean and were calculated to ensure that all data fell within the range of G_{max} . Figure 3 shows an example of the noise frequency distribution.

Adam [Kingma 2015] and AdamW [Loshchilov 2019] were used as optimizers, ReLU [Nair 2010], [Krizhevsky 2012] as the activation function, MAE and MSE as the loss functions, and WMAPE as the evaluation metric. The theoretical solution becomes zero under certain conditions, which makes ordinary error-rate calculations impossible. Therefore, WMAPE was adopted to evaluate the error rate across the entire distribution from the ratio of the sums, and this was calculated using the following Equation (5):

$$WMAPE = \frac{\sum_i^m \sum_j^k |X_{MLi,j} - X_{Theo i,j}|}{\sum_i^m \sum_j^k |X_{Theo i,j}|} \times 100\% \quad (5)$$

Hyperparameter		Value
Dataset	Step length ratio k	0~1.00
	Film thickness ratio m	1.00~2.00
	Size S	121, 441, 1 681
	Maximum noise $G_{\max}$	0, 6, 12, 18
	Generation method	Grid
Hidden layers	Number of layers L	1~5
	Number of nodes N	$2^2 \sim 2^5$
Batch size B		$2^1 \sim 2^5$
Epochs E		300
Optimization function		Adam, AdamW
Learning rate r_L		0.001~0.003
Activation function		ReLU
Loss function		MAE, MSE
Evaluation function		WMAPE

Table 1. Various hyperparameter

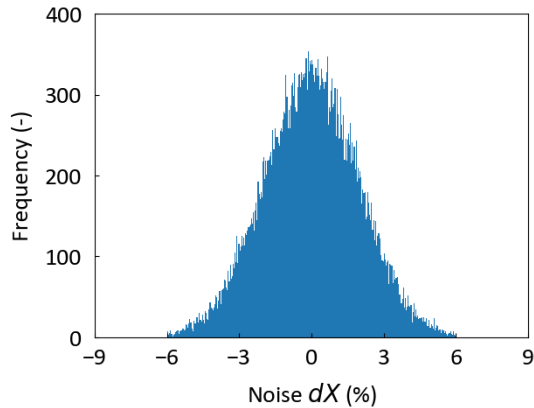


Figure 3. Example of the noise frequency distribution ($G_{\max} = 6\%$)

3 RESULTS AND DISCUSSION

3.1 Training with the theoretical solution

During the training process, confirmation of the learning curve is necessary in order to determine whether processing is proceeding correctly. Figure 4 shows an example learning curve, presenting the MSE curve for ($S = 441, L = 5, N = 32, B = 8, \text{Adam}[r_L = 0.002]$, MSE). The MSE decreases as the number of epochs increases, confirming that the training progresses properly, whereas fluctuations in the value can be observed when $E > 150$. The number of model parameters $P(= N^L)$ is large, so fitting effectiveness improved during the early stages of training. However, training likely became unstable in the latter stages due either to the increased difficulty of optimizing weight coefficients with high degrees of freedom or to overfitting. Figure 5 shows WMAPE curves comparing with predicted and theoretical solutions for each dataset during the same training

process. A qualitative trend similar to that of the MSE is observed. The correlation coefficient between MSE and WMAPE is approximately 0.9, indicating a strong proportional relationship. Typically, training applications evaluate performance using the loss function value because no theoretical solution exists, whereas the present study used WMAPE for all accuracy assessments.

Figure 6 shows the relationship between the WMAPE values of the test data and validation data for each dataset size, and Figure 7 shows the relationship between dataset size and the correlation coefficient of the WMAPE values for the two datasets. An increasing dataset size also increases the correlation coefficient, with the coefficient exceeding 0.9 at $S = 441$, indicating a strong proportional relationship between the validation error rate and the test error rate.

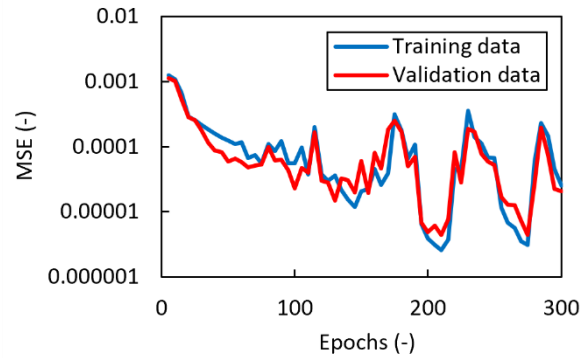


Figure 4. MSE curves ($S = 441, L = 5, N = 32, B = 8, \text{Adam}[r_L = 0.002]$)

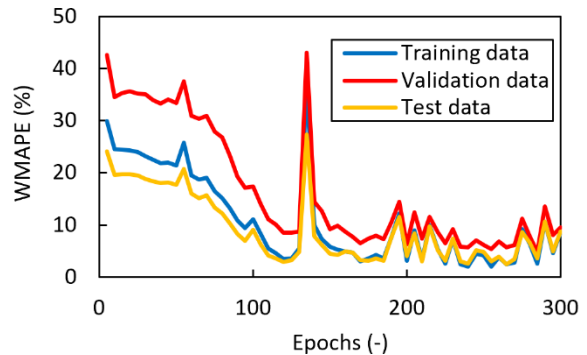


Figure 5. WMAPE curves ($S = 441, L = 5, N = 32, B = 8, \text{Adam}[r_L = 0.002]$)

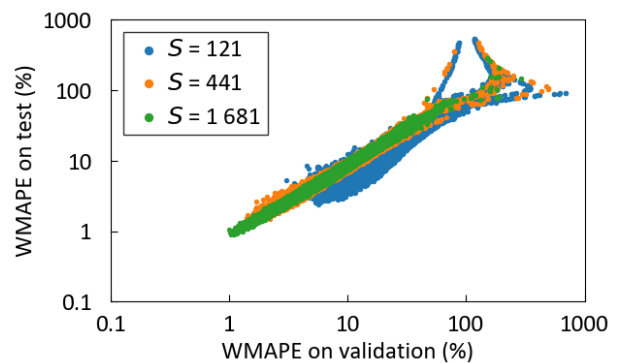


Figure 6. Relationship between the WMAPE values of the test data and validation data

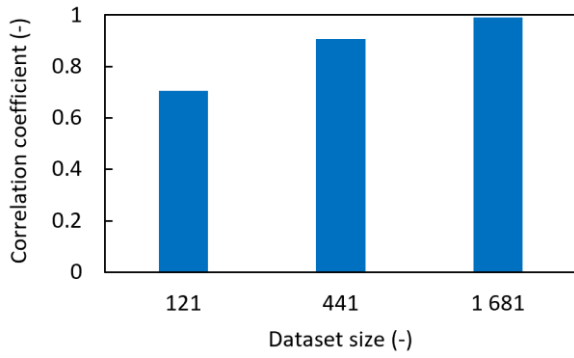


Figure 7. Relationship between dataset size and the correlation coefficient

Figure 8 presents the relationship between the ratio of the number of parameters to the dataset size and the correlation coefficient, and Figure 9 shows its relationship with WMAPE. As the ratio increases, the correlation coefficient tends to rise, and WMAPE tends to decrease. In contrast, when the dataset size is extremely large relative to the number of parameters, or when the two are approximately equal, both the correlation coefficient and approximation accuracy tend to decline. This is thought to result from underfitting or from double descent. In the present study, higher correlation coefficients and better approximation accuracy were generally observed, regardless of dataset size, when the number of parameters exceeded the dataset size by more than a factor of 10. This tendency is also consistent with the scaling laws discussed in the LLM field [Hoffmann 2022].

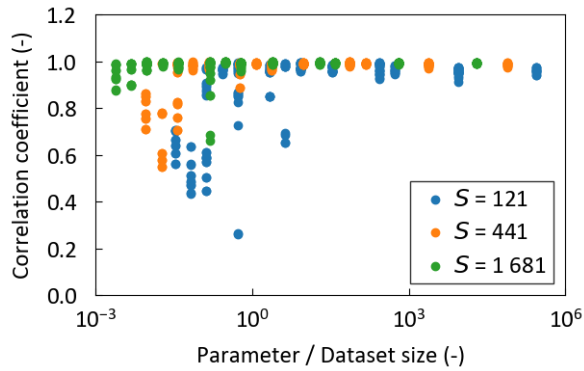


Figure 8. Relationship between the ratio of the number of parameters to the dataset size and the correlation coefficient

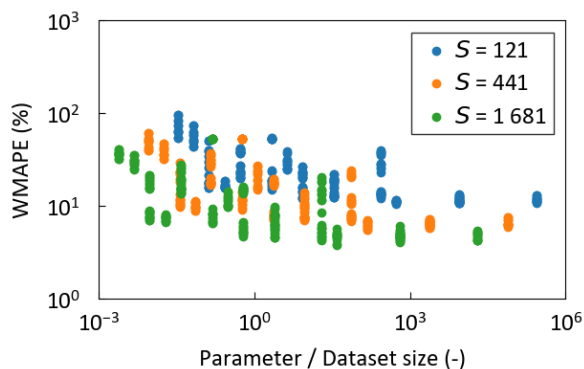


Figure 9. Relationship between the ratio and WMAPE

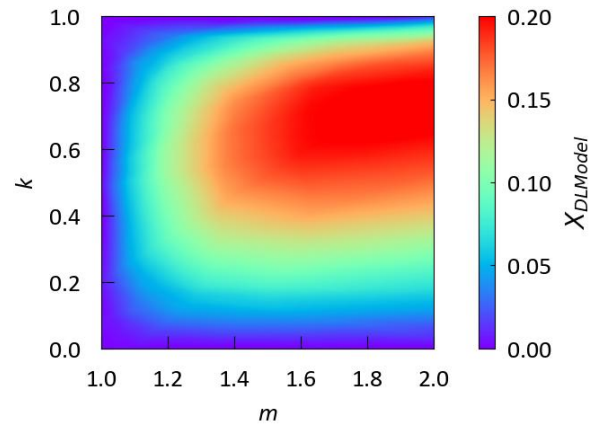


Figure 10. Predicted solution distribution ($S = 441$, $L = 5$, $N = 32$, $B = 8$, Adam[$\tau_L = 0.002$], MSE)

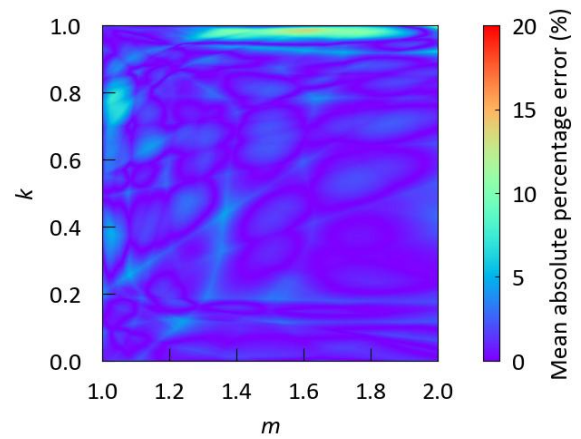


Figure 11. Distribution of the mean absolute error rate between the predicted solution and the theoretical solution

As an example, Figure 10 shows the predicted solution distribution of the trained model, and Figure 11 shows the distribution of the mean absolute error rate between the predicted solution distribution and the theoretical solution. Comparison with the theoretical solution shown in Figure 2 reveals that the model predicts the qualitative characteristics of the distribution extremely well, and the WMAPE relative to the theoretical solution was approximately 1.3%. Meanwhile, approximation accuracy deteriorates in the region $0.8 < k < 1.0$, which is thought to result from the large solution gradient, leading to a relative shortage of data compared with other regions.

3.2 Training with noisy data

Figure 12 shows the average of the post-training test results on theoretical solutions to which Gaussian noise had been added. The approximation accuracy of the predicted solutions tended to decline as the magnitude of the Gaussian noise increased, but the average WMAPE remained approximately 4.7 % even when the noise level reached 18 %. Therefore, the results suggest that essential approximation can still be achieved without substantial noise influence when operating within realistic measurement error ranges. Figure 13 shows the noise distribution for $S = 121$ and $G_{max} = 18$ %, and Figure 14 shows the distribution of the mean absolute error rate of the predicted solution relative to the theoretical solution after training on a dataset containing this noise. The decline in approximation accuracy in the region $0.8 <$

$k < 1.0$ is similar to that observed during training with the theoretical solution, and the substantial increase in the error rate was thought to be due to greater sensitivity to noise.

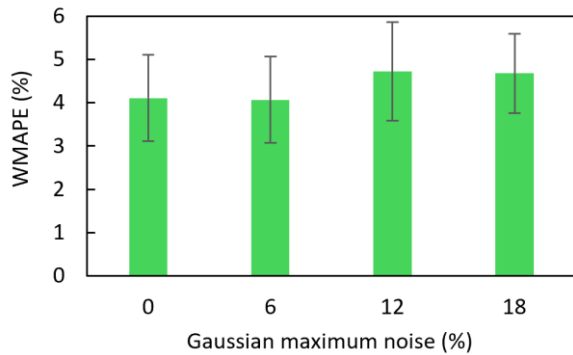


Figure 12. Relationship Gaussian noise and WMAPE

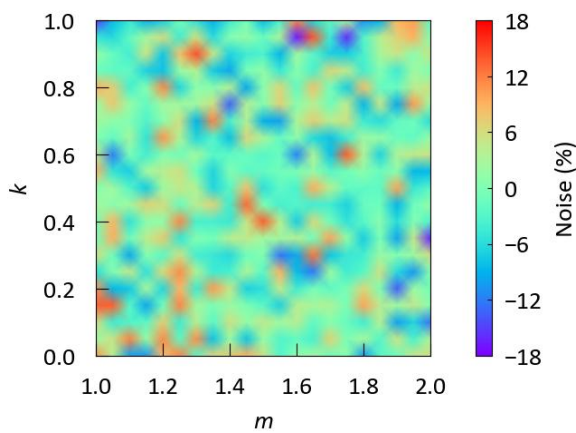


Figure 13. Noise distribution ($S = 121$, $G_{\max} = 18\%$)

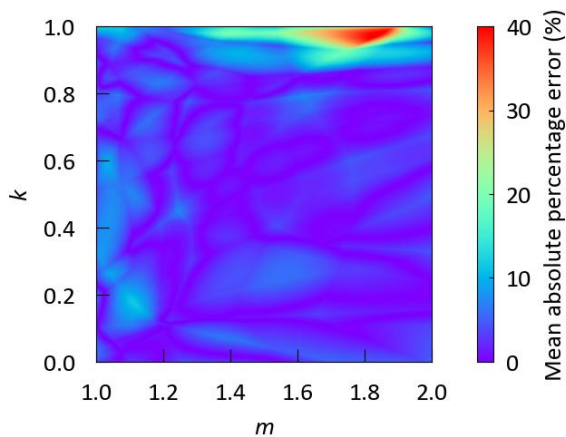


Figure 14. Distribution of the mean absolute error rate of the predicted solution relative to the theoretical solution ($S = 121$, $L = 4$, $N = 32$, $B = 16$, $\text{AdamW}[r_l = 0.001]$, MSE , $G_{\max} = 18\%$)

4 CONCLUSIONS

In this paper, a surrogate model for the Rayleigh step bearing, whose solution is theoretically known, was constructed to verify the validity and effectiveness of applying deep learning models to fluid lubrication problems. The findings obtained are summarized below.

- 1) Results presented that a sufficiently large size of dataset allowed error evaluation on validation data to be treated in the same manner as evaluation against the theoretical solution. Furthermore, when the number of parameters exceeded the dataset size by more than a factor of 10, higher correlation coefficients between test and validation accuracy, as well as better approximation accuracy, were observed regardless of dataset size. However, the required size differed depending on the target problem and therefore required careful study.
- 2) Training was conducted with restricted data sizes using a primitive method such as linear search. Nevertheless, a surrogate model with practically sufficient accuracy was obtained, with an approximation error rate of approximately 1.3 %.
- 3) Training was performed on datasets to which Gaussian noise had been intentionally added, assuming the data contained errors like those in experimental measurements. Results presented a decline in approximation accuracy with increasing noise levels, but the approximation error rate within the interpolation region remained at 4.7 % even at an error level of 18% under 3σ (three times the standard deviation) conditions. These results suggest that if the acquired values possess realistic measurement accuracy, it may be possible to construct a surrogate model that is not misled by such errors.

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